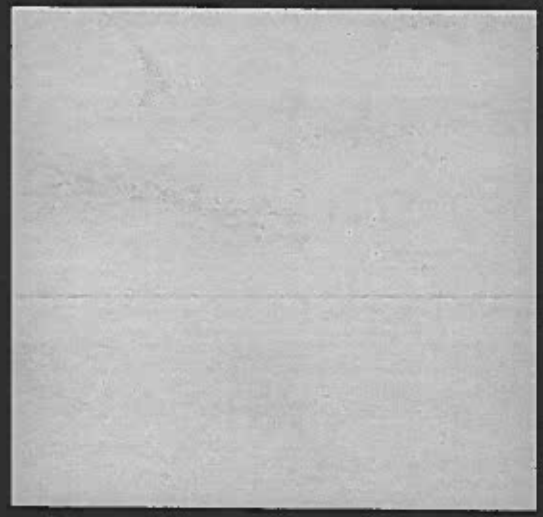
TEXAS INSTRUMENTS **SOLID STATE SOFTWARE™ CARTRIDGE**



STATISTICS

ADDENDUM

pp. 100 and 118

The computer must be in the radian mode to execute the F-Distribution and Student's t-Distribution programs. To set the radian mode, enter RAD before beginning execution of the program.

Part Number 1055926-3

TEXAS INSTRUMENTS SOLID STATE SOFTWARE™ CARTRIDGE

STATISTICS

This book was developed by:
Steven W. Smith

Programs were written by:
Jeff Gray

With contributions by:
Chris Conover
Robert E. Gahl
Kenneth E. Heichelheim
S. S. Millwright
Charles D. O'Grady, Jr.
David G. Thomas
Robert E. Whitsitt, II

Artwork and layout were coordinated and executed by:
Gaither and Davy, Inc.

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Introduction

The Statistics Library is an interchangeable *Solid State Software*™ cartridge that contains 14 professionally written, easy-to-use programs chosen to serve a broad range of statistical applications. The 14 programs require little or no programming knowledge or experience to use, and allow you to begin taking advantage of the power of your computer immediately. Other *Solid State Software* cartridges are available and can be obtained from most TI retailers or ordered directly from Texas Instruments.

Using this Manual

One section of this manual is devoted to each library program. The most important part of each section is the User Instructions which explain the operation of the program. The User Instructions have a special format designed to provide a maximum of information with a minimum of words. The first few times that you run a library program, you may wish to follow the User Instructions step-by-step. Once you are familiar with a program, you may only need to use these instructions as a reference. The extensive self-explanatory prompting built into the programs minimizes the need to carry instructions with the computer.

Examples are given for each program to illustrate its capabilities and requirements. The examples also allow you to check your cartridge by comparing your answers to those shown in this manual. Where more than one example is given in a section, it is a good practice to work the examples consecutively.

There is an appendix in the back of this manual which lists the user accessible subprograms found within the programs of this library. They contain many of the common and helpful routines which may be very useful to you when writing your own programs.

Using Library Programs

The following sections discuss the Directory/Contents program and a few general guidelines for running the programs in this library.

Directory Program

The names of the programs in this library are listed in the Directory program. It is accessed by entering **RUN "DIR"** or **RUN "CONTENTS"**. The program displays two names for each program—the long name which is the descriptive title of the program and the short name which is used to call the program for execution. For example, the long name for the first program in this library is HISTOGRAM and its short name is "HIST". To run this program, type **RUN "HIST"** and press **[ENTER]**.

There are four editing keys used to display the names in the directory.

UP arrow — displays the previous long name in the directory. If the first program name is being displayed, the directory program is exited.

DOWN arrow — displays the next long name in the directory. If the last program name is being displayed, the directory program is exited.

RIGHT arrow — displays the short name of a program if the long name is in the display. If a short name is in the display, the key is ignored.

LEFT arrow — displays the long name of a program if the short name is in the display. If a long name is in the display, the key is ignored.

User Instructions—Directory/Contents

Select the Directory program by entering **RUN "DIR"**.

Step	Display	Procedure/Comment	Goto
1.	STATISTICS LIBRARY	Cartridge name.	2
2.	Use Printer?	a. Accept printer usage by entering **Y**.	3
		b. Reject printer usage by entering **N**.	4
3.	Enter Device Name:	Enter output device name.	4
4.	Long program name	Displays long program name.	5
5.		a. To see short name of same program, press right arrow.	6
		b. To see long name of previous program, press up arrow.	4
		c. To see long name of next program, press down arrow.	4
6.	Short program name	Displays short program name.	7
7.		a. To see long name of same program, press left arrow.	4
		b. To see long name of previous program, press up arrow.	4
		c. To see long name of next program, press down arrow.	4

General Guidelines

To simplify the use of *Solid State Software*, TI has implemented a few easily remembered rules for program prompts. You should closely follow the User Instructions the first time you execute a program. Thereafter, you should be able to execute the program by following these simple guidelines.

- Use of the term "enter" has a specific meaning in the User Instructions and examples within Compact Computer library programs. When you see this term, you are to key in the value or information which is indicated by the statement and then press **[ENTER]** to execute the entry. The **[ENTER]** key is also used to display consecutive outputs, to accept current values, and to continue the program following certain prompts.
- The procedure for beginning execution of library programs is explained immediately before the User Instructions of each program. This procedure involves entering the **[RUN]** command followed by the short name for the program in quotation marks and any other information which is necessary for the execution of the particular program.
- The first step in all Compact Computer library programs is the display of the program name. The program name remains paused in the display for three seconds and then continues to the next prompt. Pressing **[ENTER]** or **[CLR]** causes the next prompt to be displayed immediately.
- Input prompts (prompts which indicate that a value must be entered) are always followed by colons. When applicable, default values are displayed immediately after the colon. You have the option of either entering a new value for the variable or accepting the current value by pressing **[ENTER]**. This feature is useful when editing input.
- When you enter a value which has less digits than the displayed value, space over the extra digits to insure that only the new value is entered.
- Prompts ending in a ? are questions requiring either yes or no responses. It is only necessary to enter a **y** or a **n** in either upper or lower case as a response.
- All commands and responses except yes and no must be entered exactly as they are presented in the User Instructions of this library. **Note:** Commands and responses in this library are represented with upper case letters. However, the Compact Computer makes no distinction between upper or lower case letters so they may be used interchangeably.

- Displayed messages such as instructions and output require you to press **[ENTER]** to proceed to the next message or prompt. Certain prompts are paused for three seconds before proceeding to the next prompt. Pressing **[ENTER]** after a paused prompt causes the next prompt to be displayed immediately.
- Any special handling of the response keys is described expressly in the User Instructions or by further prompting.
- You may use equations as numeric input in response to a prompt. For example, you may use PI/2 as an input value. When the computer is waiting for a response, you may perform arithmetical computations without affecting the program. However, the value which is in the display when **[ENTER]** is pressed is accepted as the input. This means that you may not use an equation which requires the use of **[ENTER]** in its computations.
- To halt a running program, press **[BREAK]**. Entering **CON** continues the program from the point where **[BREAK]** was pressed even if the computer is turned off after you press **[BREAK]**. If **[BREAK]** is pressed when a prompt is in the display, the prompt may not reappear after **CON** is entered although the computer is still waiting for a response to that prompt.

Using the Optional Printer

If a printer is connected to the Compact Computer and you have responded **YES** to the `Use Printer?` prompt, a printed record is produced which contains all of the information needed to duplicate a particular execution of a program (with the exception of subprograms which you enter) including: the name of the program you have chosen; the options selected; the pertinent input data; and the results.

Use of the printer is made possible by responding **YES** to the `Use Printer?` prompt and then entering the appropriate peripheral ID number in response to the `Enter Device Name:` prompt. See the peripheral *Owner's Manual* for the appropriate ID number.

The presence of the printer also alters normal program operation. Since a printed record of the results is produced, operation of the program is not stopped for viewing individual results. Instead, a continuous list of results is printed until the output is complete.

Information on connecting the printer may be found in the peripheral *Owner's Manual.*

Using the Optional *Wafertape*™ Digital Tape Drive

The *Wafertape*™ digital tape drive may be used as a means of inputting or recording large amounts of data or for recording information in order to reproduce an exact run of a program. Those programs which are designed for use with the *Wafertape* peripheral incorporate special prompts indicating when you need to input and record your data. The only information which you must provide is the device on which the data is recorded and the filename for that data.

The prompt for entry of this information is `Enter Device.Filename:`. The device must be entered as an integer from 1 through 7. The filename may be chosen by you and may contain any printable character (except control characters) but must be 12 characters or less in length.

Information on connecting the *Wafertape* digital tape drive may be found in the *Owner's Manual.*

Caring for Cartridges

Even though the cartridges for your Compact Computer are durable devices, you should handle them with care. Follow these precautions when handling the cartridges.

- **Be sure that your body is free of static electricity.** Prior to handling any cartridge, touch some metal object to discharge any static electricity you may be carrying.
- Keep the cartridge port cover secure on the computer to keep the cartridge port dust free.
- Keep the contact area of the cartridges clean. A buildup of debris or foreign particles on the contacts can impair their operation. Keep the cartridges stored either in the original container or in the computer's cartridge port.
- Use a cotton swab soaked in alcohol to clean the cartridge and cartridge port contacts when necessary. After the alcohol has dried, remove any remaining lint with a clean, soft-bristled brush.

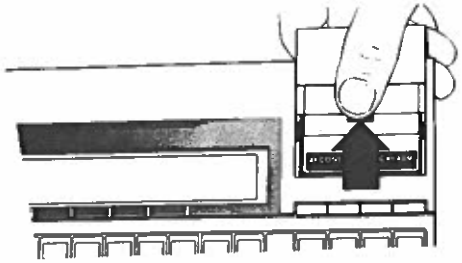
  CAUTION: Do not use any other liquid substance to clean the contacts.

Installing and Replacing Cartridges

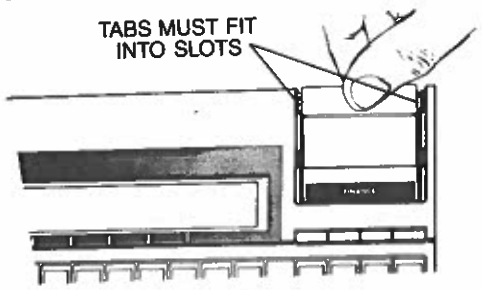
The Statistics Library cartridge can easily be installed or replaced. *Solid State Software* cartridges are installed in the cartridge port of the Compact Computer. Turn the computer off when installing or replacing a cartridge. Installing a cartridge while the computer is on may result in memory loss.

Use the following procedure when installing a cartridge.

1. Turn the computer off.
2. Slide the cartridge port cover back and remove it from the computer as shown in the diagram below.



3. If a cartridge is already installed, remove it by pushing the cartridge away from the keyboard until it is released. Lift the cartridge from the cartridge port.
4. Lay the cartridge to be installed in the cartridge port, with the cartridge name facing up and toward the keyboard.
5. Press firmly on the back of the cartridge and slide it toward the keyboard until the tabs on the cartridge enter the slots provided and the cartridge locks into place. Replace the cartridge port cover and check that its tabs also fit into the slots.



Descriptive Statistics

The programs in this section are designed to help you interpret sets of data. Statistical data is evaluated in order to make a decision. This decision usually involves choosing between two or more alternatives called hypotheses. For example, if you want to determine whether or not a coin is balanced, the hypotheses that you wish to test are:

H_0: the coin is balanced ($p = 0.5$)

against

H_1: the coin is unbalanced ($p \neq 0.5$).

H_0 is called the null hypothesis. In the experiment above, p is the probability of heads occurring on any toss of the coin.

To determine whether you should reject H_0, your first step is to obtain a sample. You can then use your sample data to derive a test statistic. Since the binomial distribution is to be used (success or failure—heads or tails) as your probability model, your test statistic for this experiment is k, where k is the number of heads occurring in n tosses of the coin.

Confidence limits for a test statistic are the usual criteria established for rejecting a hypothesis. Suppose that if you fail to reject H_0, you want to be 95% certain that you are right. To achieve this degree of confidence for the type of test described above, you would construct an acceptance region for your test statistic (k_1, k_2). These values are determined such that 95% of the area bounded by the normal curve lies between k_1 and k_2, $P(k_2 - k_1) = 0.95$ and the midpoint of the interval is the expected value of k given n and p. If your test statistic k is such that $k_1 < k < k_2$, you fail to reject H_0.

This may seem like a complicated process, and it often is. However, the programs in the "Theoretical Distributions" section make it easy to determine whether to reject your null hypothesis or not. You *never accept* your null hypothesis unless all alternatives to the null hypothesis have been rejected. That is why we say that we fail to reject the null hypothesis *instead of accepting* the null hypothesis.

Most of the programs in this library follow the same pattern of development discussed above. First, a test is described. If this test meets your needs, use the program to compute a statistic. You can then use the calculated data for an appropriate program in the "Theoretical Distributions" section to determine whether to reject your hypothesis or not.

You should realize that the coin toss test is for a special case. You tested

H_0: $p = 0.5$ against H_1: $p \neq 0.5$.

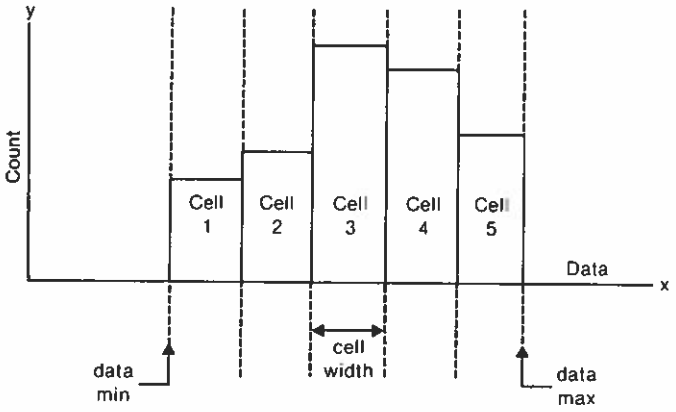
You may also test hypotheses such as

H_0: $p \leq 0.5$ against H_1: $p > 0.5$,

The program examples in this section offer the acceptance regions of the tests without proof. These examples should give you further insight into confidence levels and acceptance regions. However, a more complete discussion of confidence levels and acceptance regions is left until the "Theoretical Distributions" section of this manual.

Histogram—"HIST"

A histogram is constructed to help interpret a set of data points. Each data point is assigned to a class interval or cell of the histogram depending upon its magnitude. A typical histogram is illustrated below.



You must specify the number of cells and width of cells when using this program. The limit of the number of the cells is established by the memory capacity of your computer. With a 6K byte memory, you may enter up to 500 cells. You must also enter a lower limit for your data. Keep in mind that you should make the boundaries of your cells unattainable non-integers greater than -1. For example, if your cells contain the integer values from 0 to 3, the boundaries associated with this histogram should be $-.5$ to .5 for the first cell, .5 to 1.5 for the second cell, 1.5 to 2.5 for the third cell, and 2.5 to 3.5 for the fourth cell. These non-integers prevent any values from occurring on the boundaries. If you enter any value outside your specified histogram, the program will discard that value during histogram construction and display the error message **OUT OF RANGE ERROR**.

The cell width entered must be an integer. Keep in mind when using the binomial distribution that the following condition must hold true.

(Cell Width $\times$ ((Number of Cells) $- 1) + X_{min}$ rounded up to an integer) ≤ 7

Once you have acquired the histogram data, use this program to determine the number or frequency of counts per cell. You can also generate the means and moments of your histogram by using this program.

This program also allows you to compare the histogram constructed from your sample data points to various theoretical histograms. The theoretical histograms which you may fit are the normal distribution, binomial distribution, Poisson distribution, or the uniform distribution.

This program calculates the theoretically expected count of each cell and compiles a chi-square statistic, denoted by χ^2, for comparison to your choice of theoretical distributions (goodness of fit test) where

$$\chi^2 = \sum_{i=1}^{n} \frac{(\text{expected count} - \text{observed count})^2}{\text{expected count}}$$

with $N - 1$ degrees of freedom (N is the number of cells in your histogram). At the completion of the goodness of fit test, the program calculates $Q(\chi^2)$. The observed $Q(\chi^2)$ value is the probability that you will make a type I error, which is rejecting the null hypothesis when it is true. In order to insure the

validity of the goodness of fit test, you should collect sufficient data. When you are using the normal distribution, you should make sure that the observed count of each cell is less than five.

It is important to note that when the program calculates the theoretical histogram, the degrees of freedom lost is automatically calculated by the program. Therefore, you must insure that a sufficient number of cells exists to allow subtraction of the degrees of freedom. For example, should you accept the calculated parameters for the normal distribution, three degrees of freedom will be lost. The two additional degrees of freedom are lost because of the use of the calculated mean and standard deviation to normalize the mean of each cell.

User Instructions—Histogram

Select the Histogram program by entering **RUN "HIST"**.

Step	Display	Procedure/Comment	Goto
1.	HISTOGRAM	Program name.	2
2.	Use Printer?	a. Accept printer usage by entering **Y**.	3
		b. Reject printer usage by entering **N**.	4
3.	Enter Filename:	Enter output device name.	4
4.	Theoretical Histogram?	a. Run a goodness of fit test by entering **Y**.	5
		b. Display next option by entering **N**.	6
5.	Construct Histogram?	a. Construct histogram by entering **Y**.	6
		b. Input preconstructed histogram by entering **N**.	8
6.	Enter No. Of Elements:	Enter the maximum number of elements in your sample.	11
7.	Enter Data By Keyboard?	a. Enter data from keyboard by entering **Y**.	12
		b. Enter data from peripheral device by entering **N**.	8
8.	Enter No. Of Cells:	Enter the number of cells in your histogram.	7
9.	Enter Cell Width:	Enter width of cell boundaries.	8

(continued)

(continued)

Step	Display	Procedure/Comment	Goto
10.	Enter Minimum X Value:	Enter the minimum value for your histogram.	9
11.		a. If constructing a histogram.	12
		b. Otherwise.	31
12.		a. If data input by keyboard.	14
		b. Otherwise.	13
13.	Enter Device.Filename:	Enter the device and filename of peripheral device.	17
14.	To End Input Enter 'E'	Displays prompt. Proceed by pressing **[ENTER]**.	15
15.	Enter X(###):	Enter Xi.	16
16.		a. If more data to input.	15
		b. If no more data to input, enter **E**.	17
17.	Edit?	a. Edit data by entering **Y**.	18
		b. Accept data by entering **N**.	23
18.	Edit All Data?	a. Edit all data by entering **Y**.	19
		b. Edit single elements by entering **N**.	20
19.	Enter X(###): ####	a. Enter correct value.	23
		b. Accept current value by pressing **[ENTER]**.	23
20.	Enter Element #:	Enter the number of the element to be edited.	21
21.	Enter X(###): ####	a. Enter correct value.	22
		b. Accept current value by pressing **[ENTER]**.	22
22.	Edit More Elements?	a. Edit more elements by entering **Y**.	20
		b. Accept data by entering **N**.	23
23.		a. If you enter data from keyboard.	26
		b. Otherwise.	24

(continued)

(continued)

Step	Display	Procedure/Comment	Goto
24.	Record Data?	a. Record data by entering **Y**.	25
		b. Continue program by entering **N**.	26
25.	Enter Device.Filename:	Enter the device number and filename on which to record data.	26
26.		a. If the maximum number of elements were input.	27
		b. Otherwise.	28
27.	Input Additional Data?	a. Input additional data by entering **Y**.	12
		b. End Input by entering **N**.	28
28.	Display Cell Frequencies?	a. View frequencies of constructed histogram by entering **Y**.	29
		b. Continue program by entering **N**.	37
29.	X Cell(###)=	Displays the mean for cell i. Proceed by pressing **[ENTER]**.	30
30.	f-Cell(###)=	Displays frequency for cell i. Proceed by pressing **[ENTER]**.	38
31.	Enter f-Cell(###):	Enter frequency for cell i from previously known histogram.	32
32.	Edit?	a. Edit data by entering **Y**	33
		b. Accept data by entering **N**.	38
33.	Edit All Data?	a. Edit all data by entering **Y**.	34
		b. Edit single elements by entering **N**.	35
34.	Enter f-Cell(###): ####	a. Enter correct value.	38
		b. Accept current value by pressing **[ENTER]**.	38
35.	Enter Element #:	Enter the number of the element to be edited.	36

(continued)

(continued)

Step	Display	Procedure/Comment	Goto
36.	Enter f-Cell(###): ####	a. Enter correct value.	37
		b. Accept current value by pressing **[ENTER]**.	37
37.	Edit More Elements?	a. Edit more elements by entering **Y**.	35
		b. Accept data by entering **N**.	38
38.	Display Statistics?	a. Display means by entering **Y**.	39
		b. Continue program by entering **N**.	46
39.	Arithmetic Mean=	Displays arithmetic mean. Proceed by pressing **[ENTER]**.	40
40.	Geometric Mean=	Displays geometric mean. Proceed by pressing **[ENTER]**.	41
41.	Harmonic Mean=	Displays harmonic mean. Proceed by pressing **[ENTER]**.	42
42.	M(2)=	Displays the second moment. Proceed by pressing **[ENTER]**.	43
43.	M(3)=	Displays the third moment. Proceed by pressing **[ENTER]**.	44
44.	M(4)=	Displays the fourth moment. Proceed by pressing **[ENTER]**.	45
45.	No. Of Entries=	Displays the number of elements. Proceed by pressing **[ENTER]**.	46
46.		a. If you chose theoretical histogram.	47
		b. Otherwise.	67
47.	NORMAL DISTRIBUTION?	a. Use the normal distribution for goodness of fit by entering **Y**.	48
		b. Continue program by entering **N**.	53

(continued)

(continued)

Step	Display	Procedure/Comment	Goto
48.	Use Calculated Parameters?	a. Use computed mean and standard deviation by entering **Y**.	49
		b. Use hypothetical mean and standard deviation by entering **N**.	51
49.	Mean=	Displays computed mean. Proceed by pressing **[ENTER]**.	50
50.	Standard Dev.=	Displays computed standard deviation. Proceed by pressing **[ENTER]**.	63
51.	Enter Mean:	Enter hypothetical mean.	52
52.	Enter Standard Dev:	Enter hypothetical standard deviation.	63
53.	BINOMIAL DISTRIBUTION?	a. Use binomial distribution for goodness of fit test by entering **Y**.	54
		b. Continue program by entering **N**.	58
54.	Enter n:	Enter parameter of binomial.	55
55.	Use Calculated Parameters?	a. Accept calculated probability by entering **Y**.	56
		b. Enter hypothetical probability by entering **N**.	57
56.	p=	Displays the calculated probability. Proceed by pressing **[ENTER]**.	63
57.	Enter p:	Enter hypothetical probability.	63
58.	POISSON DISTRIBUTION?	a. Use Poisson distribution for goodness of fit test by entering **Y**.	59
		b. Continue program by entering **N**.	62
59.	Use Calculated Parameters?	a. Accept calculated mean by entering **Y**.	60
		b. Enter hypothetical mean by entering **N**.	61

(continued)

(continued)

Step	Display	Procedure/Comment	Goto
60.	m=	Displays calculated mean. Proceed by pressing **[ENTER]**.	63
61.	Enter m:	Enter hypothetical mean.	63
62.	UNIFORM DISTRIBUTION?	a. Use uniform distribution for goodness of fit test by entering **Y**.	63
		b. Continue program by entering **N**.	63
63.	Expected f-Cell(i)=	Displays expected frequency of cell i. Proceed by pressing **[ENTER]**.	64
64.	df=	Displays the degrees of freedom. Proceed by pressing **[ENTER]**.	65
65.	Chi-Square=	Displays chi-square. Proceed by pressing **[ENTER]**.	66
66.	Q(###)=	Displays chi-square statistic. Proceed by pressing **[ENTER]**.	67
67.	Exit Program?	a. Exit program by entering **Y**.	STOP
		b. Continue program by entering **N**.	68
68.		a. If you chose theoretical histogram.	69
		b. Otherwise.	4
69.	Use Same Histogram?	a. Use same histogram by entering **Y**.	47
		b. Continue program by entering **N**.	4

(continued)

Example 1

Five coins were flipped simultaneously one hundred times. The number of heads obtained during each flip were counted and recorded below.

Number of Heads	Observations
0	5
1	14
2	32
3	30
4	15
5	4

Check to see how well this data fits a binomial distribution.

Select the Histogram program by entering **RUN "HIST"**.

Step	Display	Procedure/Comment
1.	HISTOGRAM	Program name.
2.	Use Printer?	Enter **N**.
3.	Theoretical Histogram?	Enter **Y**.
4.	Construct Histogram?	Enter **N**.
5.	Enter No. Of Cells: 0	Enter **6**.
6.	Enter Cell Width: 0	Enter **1**.
7.	Enter Minimum X Value: 0	Enter **−.5**.
8.	Enter f-Cell(1): 0	Enter **5**.
9.	Enter f-Cell(2): 0	Enter **14**.
10.	Enter f-Cell(3): 0	Enter **32**.
11.	Enter f-Cell(4): 0	Enter **30**.

(continued)

(continued)

Step	Display	Procedure/Comment
12.	Enter f-Cell(5): 0	Enter **15**.
13.	Enter f-Cell(6): 0	Enter **4**.
14.	Edit?	Enter **N**.
15.	Display Statistics?	Enter **Y**.
16.	Arithmetic Mean= 2.48 Geometric Mean= 2.278944632 Harmonic Mean= 2.244668911 M(2)= 1.3696 M(3)= -.042816 M(4)= 5.01578752 No. Of Entries= 100	Displays all requested values. Proceed after each output by pressing **[ENTER]**.
17.	NORMAL DISTRIBUTION?	Enter **N**.
18.	BINOMIAL DISTRIBUTION?	Enter **Y**.
19.	Enter n: 0	Enter **5**.
20.	Use Calculated Parameters?	Enter **N**.
21.	Enter p: 0	Enter **.5**.
22.	Expected f-Cell(1)= 3.125 Expected f-Cell(2)= 15.625 Expected f-Cell(3)= 31.25 Expected f-Cell(4)= 31.25 Expected f-Cell(5)= 15.625	Displays all requested values. Proceed after each output by pressing **[ENTER]**.

(continued)

(continued)

Step	Display	Procedure/Comment
	Expected f-Cell(6)= 3.125 df= 5 Chi-Square= 1.632 Q(1.632)= .8973528953	
23.	Exit Program?	Enter **Y**.

The program calculated a $Q(\chi^2)$ value of 0.8973528953 for the binomial distribution. If you reject the hypothesis, you risk a 89.7% chance of committing a type I error.

Example 2

Construct a histogram from the data below. Use six cells with a width of two and a minimum value of 63.95.

68.4, 65.3, 66.5, 69.0, 73.6, 75.9, 69.7, 69.8, 71.0, 70.8, 67.7, 74.4, 69.9, 71.5, 71.1.

Select the Histogram program by entering **RUN "HIST"**.

Step	Display	Procedure/Comment
1.	HISTOGRAM	Program name.
2.	Use Printer?	Enter **N**.
3.	Theoretical Histogram?	Enter **N**.
4.	Enter No. Of Elements: 0	Enter **15**.
5.	Enter Data By Keyboard?	Enter **Y**.
6.	Enter No. Of Cells: 0	Enter **6**.
7.	Enter Cell Width: 0	Enter **2**.
8.	Enter Minimum X Value: 0	Enter **63.95**.
9.	To End Input Enter 'E'	Proceed by pressing **[ENTER]**.

(continued)

(continued)

Step	Display	Procedure/Comment
10.	Enter X(1): 0	Enter **68.4**.
11.	Enter X(2): 0	Enter **65.3**.
12.	Enter X(3): 0	Enter **66.5**.
13.	Enter X(4): 0	Enter **69**.
14.	Enter X(5): 0	Enter **73.6**.
15.	Enter X(6): 0	Enter **75.9**.
16.	Enter X(7): 0	Enter **69.7**.
17.	Enter X(8): 0	Enter **69.8**.
18.	Enter X(9): 0	Enter **71**.
19.	Enter X(10): 0	Enter **70.8**.
20.	Enter X(11): 0	Enter **67.7**.
21.	Enter X(12): 0	Enter **74.4**.
22.	Enter X(13): 0	Enter **69.9**.
23.	Enter X(14): 0	Enter **71.5**.
24.	Enter X(15): 0	Enter **71.1**.
25.	Edit?	Enter **N**.
26.	Record Data?	Enter **N**.
27.	Input Additional Data?	Enter **N**.
28.	See Cell Frequencies?	Enter **Y**.
29.	X Cell(1)= 64.95 f-Cell(1)= 1 X Cell(2)= 66.95 f-Cell(2)= 2	Displays all requested values. Proceed after each output by pressing **[ENTER]**.

(continued)

(continued)

Step	Display	Procedure/Comment
	X Cell(3)= 68.95 f-Cell(3)= 5 X Cell(4)= 70.95 f-Cell(4)= 4 X Cell(5)= 72.95 f-Cell(5)= 1 X Cell(6)= 74.95 f-Cell(6)= 2	
30.	Display Statistics?	Enter Y.
31.	Arithmetic Mean= 70.01666667 Geometric Mean= 69.96411141 Harmonic Mean= 69.9118111 M(2)= 7.395555556 M(3)= 4.98725902 M(4)= 140.272825 No. Of Entries= 15	Displays all requested values. Proceed after each output by pressing **[ENTER]**.
32.	Exit Program?	Enter Y.

Means and Moments—"MEAN"

The means and moments of a sample can accurately describe the shape of its distribution. For a given set of input data ($x_1, x_2, \ldots, x_n$) with associated frequencies ($f_1, f_2, \ldots, f_n$), you may use this program to calculate the means, moments, skewness, and kurtosis of a sample distribution. If $f_i = 1$ for all i, the calculations are for ungrouped data; otherwise, the calculations are for grouped data. With a 6K byte memory capacity computer, you may enter up to 300 elements for grouped data and 600 elements for ungrouped data.

In the following discussion,

$$N = \sum_{i=1}^{n} f_i$$

The arithmetic mean,

$$\bar{x}_a = (1/N) \sum_{i=1}^{n} f_i x_i$$

is simply the average value of the sample data. For ungrouped data, this value is known as the simple arithmetic mean. For grouped data it is called the weighted arithmetic mean.

The geometric mean,

$$\bar{x}_g = (10/N) \sum_{i=1}^{n} f_i \log x_i$$

is another measure of central tendency. It is especially useful in averaging ratios, percentages, and rates of change.

The harmonic mean,

$$\bar{x}_n = N \div \sum_{i=1}^{n} f_i / x_i$$

is primarily used when dealing with ratio data having physical dimensions such as miles per hour.

The rth moment about the mean (denoted by m_r) is found by the following formula.

$$m_r = (1/N) \sum_{i=1}^{n} f_i (x_i - \bar{x})^r$$

The second moment,

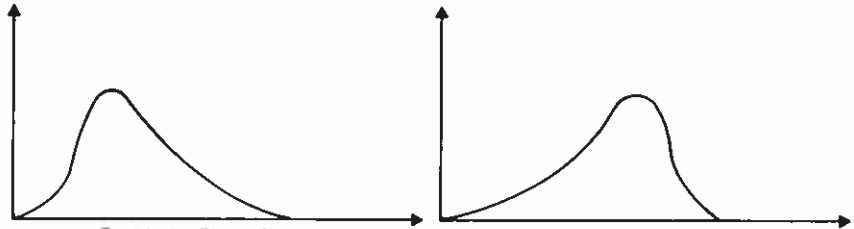
$$m_2 = 1/N \sum_{i=1}^{n} (f_i x_i - \bar{x})^2$$

is more often called the variance. It is the mean of the squared deviations of the sample data from x. This moment is used to measure the variability or dispersion of a population.

The third moment,

$$m_3 = 1/N \sum_{i=1}^{n} f_i (x_i - \bar{x})^3$$

is used to determine whether a distribution is symmetric or skewed about x. Negative and positive deviations cancel each other out since all deviations in the equation for m_3 are cubed. Therefore, m_3 equals zero when the distribution is symmetric about its mean. A distribution is said to be right or positively skewed when m_3 is positive and left or negatively skewed when m_3 is negative. You may consider your distribution to be symmetric when $-0.5 <$ skewness < 0.5. The distribution is highly skewed when the skewness exceeds ± 1. The following graphs display two skewed curves.



Positively (Right) Skewed

Negatively (Left) Skewed

You may calculate a skewness coefficient, which is a measure of relative skewness, by using the following formula with data given to you from this program.

Skewness Coefficient = $m_3/(m_2)^{3/2}$

The fourth moment,

$$m_4 = 1/N \sum_{i=1}^{n} f_i (x_i - \bar{x})^4$$

is used to interpret the flatness or peakedness of a distribution curve. You may use another relative measure known as the kurtosis of distribution to obtain this information.

Kurtosis = $m_4/(m_2)^2$

The kurtosis of a normal distribution is three. For values less than three, the curve flattens out; for values greater than three, the curve becomes more peaked.

Reference: *Standard Mathematical Tables*, The Chemical Rubber Co., 1971, pp. 554-559.

User Instructions—Means & Moments

Select the Means & Moments program by entering **RUN "MEAN"**.

Step	Display	Procedure/Comment	Goto
1.	MEANS AND MOMENTS	Program name.	2
2.	Use Printer?	a. Accept printer usage by entering **Y**.	3
		b. Reject printer usage by entering **N**.	4
3.	Enter Filename:	Enter output device name.	4
4.	Enter No. Of Elements:	Enter the maximum number of elements in your sample.	5
5.	Enter Data By Keyboard?	a. Enter data from keyboard by entering **Y**.	7
		b. Enter data from peripheral device by entering **N**.	6
6.	Is Data Grouped?	a. Select grouped data by entering **Y**.	7
		b. Select ungrouped data by entering **N**.	7
7.		a. If entering data by keyboard.	9
		b. Otherwise.	8
8.	Enter Device.Filename:	Enter the device number and filename on which data is stored.	12
9.	To End Input Enter 'E'	Displays prompt. Proceed by pressing **[ENTER]**.	10
10.	Enter X(###):	a. Enter Xi.	11
		b. Indicate end of data.	13
11.		a. If data is ungrouped.	13
		b. Otherwise.	12
12.	Enter f(###):	Enter frequency for cell i.	13
13.	Edit?	a. Edit data by entering **Y**.	14
		b. Accept data by entering **N**.	14

(continued)

(continued)

Step	Display	Procedure/Comment	Goto
14.	Edit All Data?	a. Edit all data by entering **Y**.	15
		b. Edit single elements by entering **N**.	18
15.	Enter X(###): ####	a. Enter correct value.	16
		b. Accept current value by pressing **[ENTER]**.	16
16.		a. If data is ungrouped.	23
		b. Otherwise.	17
17.	Enter f(###): ####	a. Enter correct value.	23
		b. Accept current value by pressing **[ENTER]**.	23
18.	Enter Element #:	Enter the number of the element to be edited.	19
19.	Enter X(###): ####	a. Enter correct value.	20
		b. Accept current value by pressing **[ENTER]**.	20
20.		a. If data is ungrouped.	22
		b. Otherwise.	21
21.	Enter f(###): ####	a. Enter correct value.	22
		b. Accept current value by pressing **[ENTER]**.	22
22.	Edit More Elements?	a. Edit more elements by entering **Y**.	18
		b. Accept data by entering **N**.	23
23.		a. If you enter data from keyboard.	24
		b. Otherwise.	26
24.	Record Data?	a. Record data by entering **Y**.	25
		b. Continue program by entering **N**.	26
25.	Enter Device.Filename:	Enter the device number and filename on which to record data.	26
26.		a. If you input the maximum number of elements.	27
		b. Otherwise.	28

(continued)

(continued)

Step	Display	Procedure/Comment	Goto
27.	Input Additional Data?	a. Input additional data by entering **Y**. b. End input by entering **N**.	7 28
28.	Display Means?	a. Display means by entering **Y**. b. Continue program by entering **N**.	29 32
29.	Arithmetic Mean=	Displays arithmetic mean. Proceed by pressing **[ENTER]**.	30
30.	Harmonic Mean=	Displays harmonic mean. Proceed by pressing **[ENTER]**.	31
31.	Geometric Mean=	Displays geometric mean. Proceed by pressing **[ENTER]**.	32
32.	Display Moments?	a. Display moments by entering **Y**. b. Continue program by entering **N**.	33 36
33.	M(2)=	Displays the second moment. Proceed by pressing **[ENTER]**.	34
34.	M(3)=	Displays the third moment. Proceed by pressing **[ENTER]**.	35
35.	M(4)=	Displays the fourth moment. Proceed by pressing **[ENTER]**.	36
36.	Display Statistics?	a. Display statistics by entering **Y**. b. Continue program by entering **N**.	37 40
37.	Skew=	Displays the skew. Proceed by pressing **[ENTER]**.	38
38.	Kurtosis	Displays the kurtosis. Proceed by pressing **[ENTER]**.	39
39.	No. Of Entries=	Displays the number of elements. Proceed by pressing **[ENTER]**.	40
40.	Exit Program?	a. Exit program by entering **Y**. b. Continue program by entering **N**.	STOP 4

Example 1

The following lists show the respective heights in inches of a random sample of 10 men over age 45 and their adult sons.

Fathers: 67.2, 65.0, 68.3, 69.9, 66.3, 69.7, 69.5, 72.9, 70.2, 74.1.
Sons: 68.4, 65.3, 66.5, 69.0, 73.6, 75.9, 69.7, 69.8, 71.0, 70.8, 67.7, 74.4, 69.9, 71.5, 71.1.

Compare the distribution of the heights of the fathers against that of the sons.

Select the Means & Moments program by entering **RUN "MEAN"**.

Step	Display	Procedure/Comment
1.	MEANS AND MOMENTS	Program name.
2.	Use Printer?	Enter **N**.
3.	Enter No. Of Elements: 0	Enter **10**.
4.	Enter Data By Keyboard?	Enter **Y**.
5.	Is Data Grouped?	Enter **N**.
6.	To End Input Enter 'E'	Proceed by pressing **[ENTER]**.
7.	Enter X(1): 0	Enter **67.2**.
8.	Enter X(2): 0	Enter **65**.
9.	Enter X(3): 0	Enter **68.3**.
10.	Enter X(4): 0	Enter **69.9**.
11.	Enter X(5): 0	Enter **66.3**.
12.	Enter X(6): 0	Enter **69.7**.
13.	Enter X(7): 0	Enter **69.5**.
14.	Enter X(8): 0	Enter **72.9**.
15.	Enter X(9): 0	Enter **70.2**.
16.	Enter X(10): 0	Enter **74.1**.

(continued)

(continued)

Step	Display	Procedure/Comment
17.	Edit?	Enter **N**.
18.	Record Data?	Enter **N**.
19.	Input Additional Data?	Enter **N**.
20.	Display Means?	Enter **Y**.
21.	Arithmetic Mean= 69.31	Displays arithmetic mean. Proceed by pressing **[ENTER]**.
22.	Harmonic Mean= 69.20924469	Displays harmonic mean. Proceed by pressing **[ENTER]**.
23.	Geometric Mean= 69.25951514	Displays geometric mean. Proceed by pressing **[ENTER]**.
24.	Display Moments?	Enter **Y**.
25.	M(2)= 7.0269	Displays the second moment. Proceed by pressing **[ENTER]**.
26.	M(3)= 3.938892	Displays the third moment. Proceed by pressing **[ENTER]**.
27.	M(4)= 114.132659	Displays the fourth moment. Proceed by pressing **[ENTER]**.
28.	Display Statistics?	Enter **Y**.
29.	Skew= .2114600903	Displays the skew. Proceed by pressing **[ENTER]**.
30.	Kurtosis= 2.311438747	Displays the kurtosis. Proceed by pressing **[ENTER]**.
31.	No. Of Entries= 10	Displays the number of elements. Proceed by pressing **[ENTER]**.
32.	Exit Program?	Enter **N**.
33.	Enter No. Of Elements: 10	Enter **15**.

(continued)

(continued)

Step	Display	Procedure/Comment
34.	Enter Data By Keyboard?	Enter **Y**.
35.	Is Data Grouped?	Enter **N**.
36.	To End Input Enter 'E'	Proceed by pressing **[ENTER]**.
37.	Enter X(1): 0	Enter **68.4**.
38.	Enter X(2): 0	Enter **65.3**.
39.	Enter X(3): 0	Enter **66.5**.
40.	Enter X(4): 0	Enter **69**.
41.	Enter X(5): 0	Enter **73.6**.
42.	Enter X(6): 0	Enter **75.9**.
43.	Enter X(7): 0	Enter **69.7**.
44.	Enter X(8): 0	Enter **69.8**.
45.	Enter X(9): 0	Enter **71**.
46.	Enter X(10): 0	Enter **70.8**.
47.	Enter X(11): 0	Enter **67.7**.
48.	Enter X(12): 0	Enter **77.4**.
49.	Enter X(13): 0	Enter **69.9**.
50.	Enter X(14): 0	Enter **71.5**.
51.	Enter X(15): 0	Enter **71.1**.
52.	Edit?	Enter **Y**.
53.	Edit All Data?	Enter **N**.
54.	Enter Element #: 0	Enter **12**.
55.	Enter X(12): 77.4	Enter **74.4**.
56.	Edit More Elements?	Enter **N**.

(continued)

(continued)

Step	Display	Procedure/Comment
57.	Record Data?	Enter **N**.
58.	Input Additional Data?	Enter **N**.
59.	Display Means?	Enter **Y**.
60.	Arithmetic Mean= 70.30666667	Displays arithmetic mean. Proceed by pressing **[ENTER]**.
61.	Harmonic Mean= 70.19915809	Displays harmonic mean. Proceed by pressing **[ENTER]**.
62.	Geometric Mean= 70.25279208	Displays geometric mean. Proceed by pressing **[ENTER]**.
63.	Display Moments?	Enter **Y**.
64.	M(2)= 7.609955556	Displays the second moment. Proceed by pressing **[ENTER]**.
65.	M(3)= 4.93266703	Displays the third moment. Proceed by pressing **[ENTER]**.
66.	M(4)= 152.04781	Displays the fourth moment. Proceed by pressing **[ENTER]**.
67.	Display Statistics?	Enter **Y**.
68.	Skew= .2349679456	Displays the skew. Proceed by pressing **[ENTER]**.
69.	Kurtosis= 2.625523613	Displays the kurtosis. Proceed by pressing **[ENTER]**.
70.	No. Of Entries= 15	Displays the number of elements. Proceed by pressing **[ENTER]**.
71.	Exit Program?	Enter **Y**.

Example 2

When large groups of data are involved, you may find it easier to group your data before entering it. The sample heights of the sons data from the last example is grouped below.

Range	64<X<66	66<X<68	68<X<70	70<X<72	72<X<74	74<X<76
Frequency	1	2	5	4	1	2
Assigned Value	65	67	69	71	73	75

Select the Means & Moments program by entering **RUN "MEAN"**.

Note: This example is designed for use with a storage device such as a *Wafertape* digital tape drive or cassette tape. To run this example without a storage device, respond **N** to the prompt Record Data? in step 26.

Step	Display	Procedure/Comment
1.	MEANS AND MOMENTS	Program name.
2.	Use Printer?	Enter **N**.
3.	Enter No. Of Elements: 0	Enter **10**.
4.	Enter Data By Keyboard?	Enter **Y**.
5.	Is Data Grouped?	Enter **Y**.
6.	To End Input Enter 'E'	Proceed by pressing **[ENTER]**.
7.	Enter X(1): 0	Enter **65**.
8.	Enter f(1): 0	Enter **1**.
9.	Enter X(2): 0	Enter **67**.
10.	Enter f(2): 0	Enter **3**.
11.	Enter X(3): 0	Enter **69**.
12.	Enter f(3): 0	Enter **5**.
13.	Enter X(4): 0	Enter **71**.
14.	Enter f(4): 0	Enter **4**.

(continued)

(continued)

Step	Display	Procedure/Comment
15.	Enter X(5): 0	Enter **73**.
16.	Enter f(5): 0	Enter **1**.
17.	Enter X(6): 0	Enter **75**.
18.	Enter f(6): 0	Enter **2**.
19.	Enter X(7): 0	Enter **E** to end data input.
20.	Edit?	Enter **Y**.
21.	Edit All Data?	Enter **N**.
22.	Enter Element #: 0	Enter **2**.
23.	Enter X(2): 67	Accept current value by pressing **[ENTER]**.
24.	Enter f(2): 3	Enter **2**.
25.	Edit More Elements?	Enter **N**.
26.	Record Data?	Enter **Y**.
27.	Enter Device.Filename:	Enter **"1.STATS1"** to record data.
28.	Display Means?	Enter **Y**.
29.	Arithmetic Mean= 70.06666667	Displays arithmetic mean. Proceed by pressing **[ENTER]**.
30.	Harmonic Mean= 69.96188555	Displays harmonic mean. Proceed by pressing **[ENTER]**.
31.	Geometric Mean= 70.01414879	Displays geometric mean. Proceed by pressing **[ENTER]**.
32.	Display Moments?	Enter **Y**.
33.	M(2)= 7.395555556	Displays the second moment. Proceed by pressing **[ENTER]**.

(continued)

(continued)

Step	Display	Procedure/Comment
34.	M(3)= 4.98725903	Displays the third moment. Proceed by pressing **[ENTER]**.
35.	M(4)= 140.272828	Displays the fourth moment. Proceed by pressing **[ENTER]**.
36.	Display Statistics?	Enter **Y**.
37.	Skew= .2479737556	Displays the skew. Proceed by pressing **[ENTER]**.
38.	Kurtosis= 2.564672677	Displays the kurtosis. Proceed by pressing **[ENTER]**.
39.	No. Of Entries= 15	Displays the number of elements. Proceed by pressing **[ENTER]**.
40.	Exit Program?	Enter **Y**.

Parametric Tests: Evaluation of the t-Statistic

The distribution of the t-statistic depends on the population mean, not on the population variance. As a result, this statistic is often used to compare the means of different populations. If your samples are from normal populations *with the same variance*, you can use these two programs to determine the t-statistic and the corresponding number of degrees of freedom. You may then use the Students' t-Distribution program to complete your comparison of the population means.

t-Test: Paired Observations—"TTESTP"

You can use this program to evaluate the t-statistic

$$t = \bar{\Delta}\sqrt{n}\,/s_{\Delta}$$

with $n-1$ degrees of freedom in order to test the hypothesis that two normally distributed populations with the same unknown variance have the same mean. This test is performed using n paired observations from the two samples. Up to 300 elements may be entered with a 6K byte memory capacity computer. In the above equation,

$\bar{\Delta}$ = the mean of the differences between the paired values and is found by the formula:

$$\bar{\Delta} = \frac{\sum_{i=1}^{n} (x_i - y_i)}{n}$$

n = the sample size

$s_{\bar{\Delta}}$ = the standard deviation of the differences between the paired values using $n-1$ degree of freedom and is computed by the formula:

$$s_{\bar{\Delta}} = \sqrt{\frac{n\sum_{i=1}^{n} (x_i - y_i)^2 - \left(\sum_{i=1}^{n} (x_i - y_i)\right)^2}{n-1}}$$

User Instructions—t-Test: Paired Observations

Select the Paired t-test program by entering **RUN "TTESTP"**.

Step	Display	Procedure/Comment	Goto
1.	t-TEST:PAIRED	Program name.	2
2.	Use Printer?	a. Accept printer usage by entering **Y**.	3
		b. Reject printer usage by entering **N**.	4
3.	Enter Filename:	Enter output device name.	4
4.	Enter Data By Keyboard?	a. Enter data from keyboard by entering **Y**.	5
		b. Enter data from peripheral device by entering **N**.	5
5.	Enter No. Of Elements:	Enter the maximum number of elements in your sample.	6
6.		a. If you enter data from keyboard.	8
		b. Otherwise.	7
7.	Enter Device.Filename:	Enter the device and filename of peripheral device.	11
8.	To End Input Enter 'E'	Displays prompt. Proceed by pressing **[ENTER]**.	9
9.	Enter X(###):	a. Enter X value.	10
		b. Indicate end of data.	11
10.	Enter Y(###):	Enter X's paired Y value.	11
11.	Edit?	a. Edit data by entering **Y**.	12
		b. Accept data by entering **N**.	13
12.	Edit All Data?	a. Edit all data by entering **Y**.	13
		b. Edit single elements by entering **N**.	15
13.	Enter X(###): ####	a. Enter correct value.	14
		b. Accept current value by pressing **[ENTER]**.	14
14.	Enter Y(###): ####	a. Enter correct value.	19
		b. Accept current value by pressing **[ENTER]**.	19

(continued)

(continued)

Step	Display	Procedure/Comment	Goto
15.	Enter Element #:	Enter the number of the particular elements to be edited.	16
16.	Enter X(###): ####	a. Enter correct value.	17
		b. Accept current value by pressing **[ENTER]**.	17
17.	Enter Y(###): ####	a. Enter correct value.	18
		b. Accept current value by pressing **[ENTER]**.	18
18.	Edit More Elements?	a. Edit more elements by entering **Y**.	15
		b. Accept data by entering **N**.	19
19.		a. If you enter data from keyboard.	20
		b. Otherwise.	22
20.	Record Data?	a. Record data by entering **Y**.	21
		b. Continue program by entering **N**.	22
21.	Enter Device.Filename:	Enter the device number and filename on which to record data.	14
22.		a. If you input the maximum number of elements.	23
		b. Otherwise.	24
23.	Input Additional Data?	a. Input additional data by entering **Y**.	6
		b. End input by entering **N**.	24
24.	Standard Dev=	Displays standard deviation. Proceed by pressing **[ENTER]**.	25
25.	Delta X=	Displays average distance between pairs. Proceed by pressing **[ENTER]**.	26
26.	t=	Displays t-statistic. Proceed by pressing **[ENTER]**.	27
27.	df=	Displays degrees of freedom. Proceed by pressing **[ENTER]**.	28
28.	Exit Program?	a. Exit program by entering **Y**.	STOP
		b. Continue program by entering **N**.	4

Example

In an experiment to compare two different diets for pigs, a farmer randomly selects a pair of pigs from each of ten litters. He then chooses one pig from each pair and places them on diet A for a fixed period of time. The other pig from each pair is placed on diet B during the same period. At the end of the experiment, the farmer weighs each pig to see how much it has gained. The results are tabulated below.

	1	2	3	4	5	6	7	8	9	10
Diet A	21.5	18.0	14.7	19.3	21.7	22.9	22.3	19.1	13.3	19.8
Diet B	14.7	16.1	15.2	14.6	17.5	15.6	20.8	20.3	12.0	20.9

Evaluate the t-statistic for paired observations to test the hypothesis

H_0: the diets cause the same average weight gain (Ma = Mb)

against

H_1: the diets cause different average weight gains (Ma ≠ Mb)

at the 95% confidence level.

Select the Paired t-test program by entering **RUN "TTESTP"**.

Step	Display	Procedure/Comment
1.	`t-TEST:PAIRED`	Program name.
2.	`Use Printer?`	Enter **N**.
3.	`Enter Data By Keyboard?`	Enter **Y**.
4.	`Enter No. Of Elements: 0`	Enter **10**.
5.	`To End Input Enter 'E'`	Proceed by pressing **[ENTER]**.
6.	`Enter X(1): 0`	Enter **21.5**.
7.	`Enter Y(1): 0`	Enter **14.7**.
8.	`Enter X(2): 0`	Enter **18**.
9.	`Enter Y(2): 0`	Enter **16.1**.

(continued)

(continued)

Step	Display	Procedure/Comment
10.	`Enter X(3): 0`	Enter **14.7**.
11.	`Enter Y(3): 0`	Enter **15.2**.
12.	`Enter X(4): 0`	Enter **19.3**.
13.	`Enter Y(4): 0`	Enter **14.6**.
14.	`Enter X(5): 0`	Enter **21.7**.
15.	`Enter Y(5): 0`	Enter **17.5**.
16.	`Enter X(6): 0`	Enter **22.9**.
17.	`Enter Y(6): 0`	Enter **15.6**.
18.	`Enter X(7): 0`	Enter **22.3**.
19.	`Enter Y(7): 0`	Enter **20.8**.
20.	`Enter X(8): 0`	Enter **19.1**.
21.	`Enter Y(8): 0`	Enter **20.3**.
22.	`Enter X(9): 0`	Enter **13.3**.
23.	`Enter Y(9): 0`	Enter **12**.
24.	`Enter X(10): 0`	Enter **19.8**.
25.	`Enter Y(10): 0`	Enter **20.9**.
26.	`Edit?`	Enter **N**.
27.	`Record Data?`	Enter **N**.
28.	`Input Additional Data?`	Enter **N**.

29.	`Standard Dev= 3.121769441`
	`Delta X= 2.49`
	`t= 2.522310351`
	`df= 9`
		Displays all requested values. Proceed after each output by pressing **[ENTER]**.

30.	`Exit Program?`	Enter **Y**.

t-Test: Unpaired Observations—"TTESTU"

You may use this program to evaluate the t-statistic

$$t = \frac{\bar{x} - \bar{y} - \Delta}{\left(\frac{1}{n_x} + \frac{1}{n_y}\right)^{1/2} \left(\frac{\sum_{i=1}^{n_x} x_i^2 - n_y \bar{x}^2 + \sum_{i=1}^{n_y} y_j^2 - n_y \bar{y}^2}{n_x + n_y - 2}\right)^{1/2}}$$

where

Δ = the difference between the means of the X and Y populations.

n_x = the number of X observations.

n_y = the number of Y observations.

This formula uses $n_x - n_y - 2$ degrees of freedom, because the data is not paired. With a 6K byte memory capacity computer, the total number of elements which you may enter may not exceed 600, i.e., $n_x + n_y < 600$.

By using this program, you may test the hypothesis that the difference between the means of two normally distributed populations having the same unknown variance is Δ.

User Instructions—t-Test: Unpaired Observations

Select the Unpaired t-test program by entering **RUN "TTESTU"**.

Step	Display	Procedure/Comment	Goto
1.	t-TEST:UNPAIRED	Program name.	2
2.	Use Printer?	a. Accept printer usage by entering **Y**.	3
		b. Reject printer usage by entering **N**.	4
3.	Enter Filename:	Enter output device name.	4
4.	Enter Data By Keyboard?	a. Enter data from keyboard by entering **Y**.	5
		b. Enter data from peripheral device by entering **N**.	5
5.	Is Data Grouped?	a. Select grouped data by entering **Y**.	6
		b. Select ungrouped data by entering **N**.	6
6.	Input X Values...	Displays prompt. Proceed by pressing **[ENTER]**.	7
7.	Enter No. Of Elements:	Enter the maximum number of elements in sample one.	8
8.		a. If data entered by keyboard.	10
		b. Otherwise.	9
9.	Enter Device.Filename:	Enter the device number and filename on which data is stored.	13
10.	To End Input Enter 'E'	Displays prompt. Proceed by pressing **[ENTER]**.	11
11.	Enter X(###):	a. Enter Xi.	12
		b. Indicate end of data.	13
12.		a. If data is ungrouped.	14
		b. Otherwise.	13
13.	Enter f(###):	Enter frequency for cell i.	14
14.	Edit?	a. Edit data by entering **Y**	14
		b. Accept data by entering **N**.	21

(continued)

(continued)

Step	Display	Procedure/Comment	Goto
15.	`Edit All Data?`	a. Edit all data by entering **Y**.	16
		b. Edit single elements by entering **N**.	19
16.	`Enter X(###): ####`	a. Enter correct value.	17
		b. Accept current value by pressing **[ENTER]**.	17
17.		a. If data is ungrouped.	21
		b. Otherwise.	18
18.	`Enter f(###): ####`	a. Enter correct value.	21
		b. Accept current value by pressing **[ENTER]**.	21
19.	`Enter Element #:`	Enter the number of the particular element to be edited.	20
20.	`Enter X(###): ####`	a. Enter correct value.	21
		b. Accept current value by pressing **[ENTER]**.	21
21.		a. If data is ungrouped.	23
		b. Otherwise.	22
22.	`Enter f(###): ####`	a. Enter correct value.	23
		b. Accept current value by pressing **[ENTER]**.	23
23.	`Edit More Elements?`	a. Edit more elements by entering **Y**.	19
		b. Accept data by entering **N**.	24
24.		a. If data entered by keyboard.	25
		b. Otherwise.	27
25.	`Record Data?`	a. Record data by entering **Y**.	26
		b. Continue program by entering **N**.	27
26.	`Enter Device.Filename:`	Enter the device number and filename on which to record data.	27

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(continued)

Step	Display	Procedure/Comment	Goto
27.		a. If you input the maximum number of elements.	28
		b. Otherwise.	29
28.	`Input Additional Data?`	a. Input additional data by entering **Y**.	8
		b. End input by entering **N**.	29
29.	`Input Y Values...`	Displays prompt. Proceed by pressing **[ENTER]**.	30
30.	`Enter No. Of Elements:`	Enter the maximum number of elements in sample two.	31
31.		a. If data entered by keyboard.	33
		b. Otherwise.	32
32.	`Enter Device.Filename:`	Enter the device number and filename on which data is stored.	37
33.	`To End Input Enter 'E'`	Displays prompt. Proceed by pressing **[ENTER]**.	34
34.	`Enter Y(###):`	a. Enter Yi.	35
		b. Indicate end of data.	36
35.		a. If data is ungrouped.	37
		b. Otherwise.	36
36.	`Enter f(###):`	Enter frequency for cell i.	37
37.	`Edit?`	a. Edit data by entering **Y**.	38
		b. Accept data by entering **N**.	47
38.	`Edit All Data?`	a. Edit all data by entering **Y**.	39
		b. Edit single elements by entering **N**.	42
39.	`Enter Y(###): ####`	a. Enter correct value.	40
		b. Accept current value by pressing **[ENTER]**.	40
40.		a. If data is ungrouped.	47
		b. Otherwise.	41

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Step	Display	Procedure/Comment	Goto
41.	`Enter f(###): ####`	a. Enter correct value.	47
		b. Accept current value by pressing **[ENTER]**.	47
42.	`Enter Element #:`	Enter the number of the particular element to be edited.	43
43.	`Enter Y(###): ####`	a. Enter correct value.	44
		b. Accept current value by pressing **[ENTER]**.	44
44.		a. If data is ungrouped.	46
		b. Otherwise.	45
45.	`Enter f(###): ####`	a. Enter correct value.	46
		b. Accept current value by pressing **[ENTER]**.	46
46.	`Edit More Elements?`	a. Edit more elements by entering **Y**.	42
		b. Continue program by entering **N**.	47
47.		a. If data entered by keyboard.	48
		b. Otherwise.	50
48.	`Record Data?`	a. Record data by entering **Y**.	49
		b. Continue program by entering **N**.	50
49.	`Enter Device.Filename:`	Enter the device number and filename on which to record data.	50
50.		a. If you input the maximum number of elements.	51
		b. Otherwise.	52
51.	`Input Additional Data?`	a. Input additional data by entering **Y**.	31
		b. End input by entering **N**.	52
52.	`Enter Hyp. Delta Mean:`	Enter the hypothesized mean difference.	53
53.	`t=`	Displays t-statistic. Proceed by pressing **[ENTER]**.	54

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Step	Display	Procedure/Comment	Goto
54.	`df=`	Displays degrees of freedom. Proceed by pressing **[ENTER]**.	55
55.	`nx=`	Displays number of X entries. Proceed by pressing **[ENTER]**.	56
56.	`ny=`	Displays number of Y entries. Proceed by pressing **[ENTER]**.	57
57.	`Exit Program?`	a. Exit program by entering **Y**.	STOP
		b. Continue program by entering **N**.	4

Example

Two groups of patients at a major hospital are selected for an experiment to compare two drugs used for relief of pain. One group is given drug x; the other group is given drug y. The resulting number of hours of relief for each patient is given below.

Drug x: 2, 6, 4, 13, 5, 8, 4, 6.

Drug y: 6, 4, 4, 1, 8, 2, 12, 1, 5, 2.

Test the null hypothesis:

H_0: $\bar{x} - \bar{y} = 2$ hours

H_1: $\bar{x} - \bar{y} \neq 2$ hours.

Select the Unpaired t-test program by entering **RUN "TTESTU"**.

Step	Display	Procedure/Comment
1.	t-TEST:UNPAIRED	Program name.
2.	Use Printer?	Enter **N**.
3.	Enter Data By Keyboard?	Enter **Y**.
4.	Is Data Grouped?	Enter **N**.
5.	Input X Values...	Proceed by pressing **[ENTER]**.
6.	Enter No. Of Elements: 0	Enter **10**.
7.	To End Input Enter 'E'	Proceed by pressing **[ENTER]**.
8.	Enter X(1): 0	Enter **2**.
9.	Enter X(2): 0	Enter **6**.
10.	Enter X(3): 0	Enter **4**.
11.	Enter X(4): 0	Enter **13**.
12.	Enter X(5): 0	Enter **5**.
13.	Enter X(6): 0	Enter **8**.
14.	Enter X(7): 0	Enter **4**.
15.	Enter X(8): 0	Enter **6**.
16.	Enter X(9): 0	Enter **E** to end data input.
17.	Edit?	Enter **N**.
18.	Record Data?	Enter **N**.
19.	Input Y Values...	Proceed by pressing **[ENTER]**.
20.	Enter No. Of Elements: 0	Enter **12**.
21.	To End Input Enter 'E'	Proceed by pressing **[ENTER]**.
22.	Enter Y(1): 0	Enter **6**.
23.	Enter Y(2): 0	Enter **4**.

(continued)

(continued)

Step	Display	Procedure/Comment
24.	Enter Y(3): 0	Enter **4**.
25.	Enter Y(4): 0	Enter **1**.
26.	Enter Y(5): 0	Enter **8**.
27.	Enter Y(6): 0	Enter **2**.
28.	Enter Y(7): 0	Enter **12**.
29.	Enter Y(8): 0	Enter **1**.
30.	Enter Y(9): 0	Enter **5**.
31.	Enter Y(10): 0	Enter **2**.
32.	Enter Y(11): 0	Enter **E** to end data input.
33.	Edit?	Enter **N**.
34.	Record Data?	Enter **N**.
35.	Enter Hyp. Delta Mean: 0	Enter **2**.
36.	t=-.3087445631 df= 16 nx= 8 ny= 10	Displays all requested values. Proceed after each output by pressing **[ENTER]**.
37.	Exit Program?	Enter **Y**.

Non-Parametric Tests—Contingency Table Analysis—"CTBL"

A contingency table is a table which classifies each observation in two or more ways. The data given below represent the number of drivers in various age groups who were involved in zero, one, two, and more than two automobile accidents over a period of three years.

		Age of Driver					
		21-30	31-40	41-50	51-60	61-70	$N_{i\bullet}$
Number of Accidents Per Driver	0	748	821	786	720	672	3747
	1	74	60	51	66	55	306
	2	31	25	22	16	15	109
	2	9	10	6	5	7	37
	$N_{\bullet j}$	862	916	865	807	749	4199

A table in which only two classifications are considered is known as a two-way contingency table. In analyzing such a table, you are often interested in testing the hypothesis that the two classifications are independent of one another. In this example, you would like to know if the age of the driver and the number of accidents he is involved in are related. The alternative hypothesis states that the two factors are not related.

You may use the Contingency Table Analysis program to apply the chi-square test of independence to the row and column classifications of a contingency table with R rows and C columns. This program computes the chi-square statistic, denoted by χ^2, with $(R-1)(C-1)$ degrees of freedom (where R is the number of rows and C is the number of columns) and the cumulative distribution function of χ^2, $P(\chi^2)$. You fail to reject your null hypothesis whenever $P(\chi^2)$ is less than or equal to your specified confidence level.

Method Used—Contingency Table Analysis Program

Calculations are based upon χ^2 where

$$\chi^2 = \sum_{i=1}^{R} \sum_{j=1}^{C} (N_{ij} - E_{ij})^2 / E_{ij}$$

with $(R-1)(C-1)$ degrees of freedom (where R is the number of rows and C is the number of columns). In the above, N_{ij} is the count, or number of observations occurring in $cell_{ij}$ (row i, column j) of the table. E_{ij} is the maximum likelihood estimator of the number of observations that should occur in cell ij when the classifications are independent (when the row and column factors are not related). In other words, the probability of an event falling in both row i and column j is the probability of the event falling in row i multiplied by the probability that it falls in column j. This may be expressed as

$$E_{ij} = (N_{i\bullet}\ N_{\bullet j})/n$$

where

$N_{i\bullet}$ = the total number of counts in the ith row

$N_{\bullet j}$ = the total number of counts in the jth column

n = the total number of observations, $N_{i\bullet}$ or $N_{\bullet j}$

Note: With a 6K byte memory capacity computer, the total number of elements of the $N_{i\bullet}$ times $N_{\bullet j}$ must not be greater than 500.

Reference: *Practical Non-Parametric Statistics*, Conover, Wiley, 1971, pp. 154-155.

User Instructions—Contingency Table

Select the Contingency Table program by entering **RUN "CTBL"**.

Step	Display	Procedure/Comment	Goto
1.	`CONTINGENCY TABLE`	Program name.	2
2.	`Use Printer?`	a. Accept printer usage by entering **Y**.	3
		b. Reject printer usage by entering **N**.	4
3.	`Enter Filename:`	Enter output device name.	4
4.	`Enter Data By Keyboard?`	a. Enter data from keyboard by entering **Y**.	5
		b. Enter data from peripheral device by entering **N**.	5
5.	`Enter No. Of Rows:`	Enter the number of rows in your sample.	6
6.	`Enter No. Of Columns:`	Enter the number of columns in your sample.	7
7.		a. If you enter data from keyboard.	9
		b. Otherwise.	8
8.	`Enter Device.Filename:`	Enter the device number and filename on which data is stored.	10
9.	`Enter X(##,##):`	Enter Xij.	10
10.	`Edit?`	a. Edit data by entering **Y**.	11
		b. Accept data by entering **N**.	17
11.	`Edit All Data?`	a. Edit all data by entering **Y**.	12
		b. Edit single elements by entering **N**.	13
12.	`Enter X(##,##): ####`	a. Enter correct value.	13
		b. Accept current value by pressing **[ENTER]**.	13
13.	`Enter Row Number:`	Enter the number of the row you want to edit.	14
14.	`Enter Column Number:`	Enter the number of the column you want to edit.	15
15.	`Enter X(##,##): ####`	a. Enter correct value.	16
		b. Accept current value by pressing **[ENTER]**.	16

(continued)

(continued)

Step	Display	Procedure/Comment	Goto
16.	Edit More Elements?	a. Edit more values by entering **Y**. b. Accept data by entering **N**.	13 17
17.		a. If you enter data from keyboard. b. Otherwise.	18 20
18.	Record Data?	a. Record data by entering **Y**. b. Continue program by entering **N**.	19 20
19.	Enter Device.Filename:	Enter the device number and filename on which to record data.	20
20.	Chi-Square=	Displays the chi-square statistic. Proceed by pressing **[ENTER]**.	21
21.	df=	Displays degrees of freedom. Proceed by pressing **[ENTER]**.	22
22.	P(##)=	Displays area of the curve to the to the left of your chi-square. Proceed by pressing **[ENTER]**.	23
23.	Exit Program?	a. Exit program by entering **Y**. b. Continue program by entering **N**.	STOP 4

Example

200 voters in a local bond election are randomly selected and asked their opinion on an issue. The voters are then classified according to their answers to the question and whether or not they are property owners as illustrated below.

	For	Against	Undecided	$N_{j\bullet}$
Property Owner	45	39	21	105
Non-Property Owner	47	26	22	95
$N_{\bullet j}$	92	65	43	200

Test the null hypothesis that a voter's opinion of the bond issue is independent of his status as a property owner. Use a 90% confidence level.

Select the Contingency Table program by entering **RUN "CTBL"**.

Step	Display	Procedure/Comment
1.	CONTINGENCY TABLE	Program name.
2.	Use Printer?	Enter **N**.
3.	Enter Data By Keyboard?	Enter **Y**.
4.	Enter No. Of Rows: 0	Enter **2**.
5.	Enter No. Of Columns: 0	Enter **3**.
6.	Enter X(1,1): 0	Enter **46**.
7.	Enter X(1,2): 0	Enter **39**.
8.	Enter X(1,3): 0	Enter **21**.
9.	Enter X(2,1): 0	Enter **47**.
10.	Enter X(2,2): 0	Enter **26**.
11.	Enter X(2,3): 0	Enter **22**.
12.	Edit?	Enter **Y**.
13.	Edit All Data?	Enter **N**.
14.	Enter Row Number: 0	Enter **1**.
15.	Enter Column Number: 0	Enter **1**.
16.	Enter X(1,1): 46	Enter **45**.
17.	Edit More Elements?	Enter **N**.
18.	Record Data?	Enter **N**.
19.	Chi-Square= 2.172164486	Displays the chi-square statistic. Proceed by pressing **[ENTER]**.

(continued)

(continued)

Step	Display	Procedure/Comment
20.	df= 2	Displays degrees of freedom. Proceed by pressing **[ENTER]**.
21.	P(2.172164486)= .6624637081	Displays area of the curve to the left of your chi-square. Proceed by pressing **[ENTER]**.
22.	Exit Program?	Enter **Y**.

The range of chi-square is from zero to infinity. In order to evaluate this statistic, you would normally have to determine an acceptance region for the computed statistic. χ^2 depends upon the number of degrees of freedom of your test statistic and the confidence interval at which you are testing. At the 90% confidence level, this interval would be (0,4.6)* for a χ^2 with 2 degrees of freedom. Since the value calculated above for χ^2 falls within this interval, you fail to reject the null hypothesis. In other words there is no evidence that a voter's opinion is influenced by the fact the he is a property owner or not.

Note that this program also computes $P(\chi^2)$. This chi value ranges over the interval (0,1). When testing at the 90% confidence level, $P(\chi^2) = 0.90$. Thus, this test fails to reject H_0 because $P(\chi^2) \leq 0.90$.

* This value may be verified by using the Chi-Square program in the "Theoretical Distributions" section of this library.

Mann-Whitney Rank-Sum Test—"RSUM"

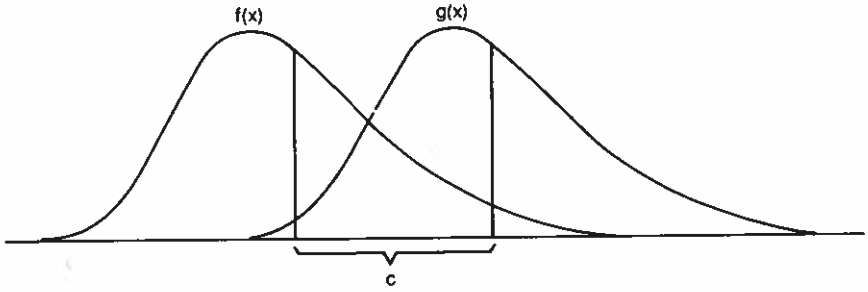
The Mann-Whitney Rank-Sum test is a procedure used to compare the means of two populations having the same distribution. You do not have to know the form of the distribution to use this test. However, assuming that the two distributions of the cumulative density functions f(x) and g(x) take the same form, you may use this program to test

H_0: the means of the populations are equal; $f(x) = g(x)$

against

H_1: the means of the populations differ by an unknown constant c, such that $f(x + c) = g(x)$.

The following graph illustrates H_1, as g(x) has shifted to the right by an amount c.



Suppose that X is a random sample of size m and Y is a random sample of size n (with a 6K byte memory capacity computer, the combined size of sample N + M must not be greater than 550 elements). Assuming that these samples are from populations having the same distribution, you may use this program to compare their means. The first step is to order the values in your two samples from smallest to largest. (The program does this for you.) For example, if your samples were

$X = (2, 5, 6)$ and $Y = (3, 6, 7)$,

the combined ordered set would be

$(x_1, y_1, x_2, x_3, y_2, y_3)$.

The assigned ranks would be (1, 2, 3, 4.5, 4.5, 6) respectively. Naturally, you should use a larger sample than this example.

When H_1 is true for a positive c, the y values tend to be larger than the x values. A statistic taking advantage of this fact is the sum of the ranks of the y's in the combined ordered set. That is, if $R(y_i)$ is the rank of y_i and T_y is the sum of the y ranks, then

$$T_y = \sum_{i=1}^{n} R(y_i)$$

In the above example, $T_y = 2 + 4.5 + 6 = 12.5$.

Now, if H_0 is true and m and n are both greater than 9 (where m = number of x values and n = number of y values), then T_y possesses an approximately normal distribution. Consequently, the Mann-Whitney statistic for y is given by the formula

$$w_y = mn - T_y + n(n+1)/2$$

and is also nearly normally distributed. The mean and variance of w_y are found by the formulas

$$w = mn/2$$

$$s_w^2 = mn(m+n+1)/12$$

The program converts the standard deviation to standard form, yielding

$$z_y = (w_y - w)/s_w$$

where z is the standard deviate for that particular Mann-Whitney statistic. You may now use this data in the Normal Distribution program and perform a lower-tailed test to evaluate your hypothesis.

If you suspect that H_1 may be true for a negative c, you should use T_x as the basis of your test. Calculations proceed similarly to the one above except that an upper-tailed test is used in the final evaluation.

Reference: *Practical Non-Parametric Statistics*, Conover, Wiley, 1971, pp. 224-229.

User Instructions—Mann-Whitney Rank Sum Test

Select the Rank Sum Test program by entering **RUN "RSUM"**.

Step	Display	Procedure/Comment	Goto
1.	RANK SUM TEST	Program name.	2
2.	Use Printer?	a. Accept printer usage by entering **Y**.	3
		b. Reject printer usage by entering **N**.	4
3.	Enter Filename:	Enter output device name.	4
4.	Enter Data By Keyboard?	a. Enter data from keyboard by entering **Y**.	5
		b. Enter data from peripheral device by entering **N**.	5
5.	Enter Size For Sample 1:	Enter the number of elements in sample one.	6
6.	Enter Size For Sample 2:	Enter the number of elements in sample two.	7
7.	Input Sample One...	Displays prompt. Proceed by pressing **[ENTER]**.	8
8.		a. If you enter data from keyboard.	10
		b. Otherwise.	9
9.	Enter Device.Filename:	Enter the device number and filename on which data is stored.	12
10.	Enter X(###):	a. Enter Xi.	11
		b. Indicate end of data.	11
11.	Edit?	a. Edit data by entering **Y**.	12
		b. Accept data by entering **N**.	17
12.	Edit All Data?	a. Edit all data by entering **Y**.	13
		b. Edit single elements by entering **N**.	14
13.	Enter X(###): ####	a. Enter correct value.	17
		b. Accept current value by pressing **[ENTER]**.	17
14.	Enter Element #:	Enter the number of the element to be edited.	16

(continued)

(continued)

Step	Display	Procedure/Comment	Goto
15.	`Enter X(###): ####`	a. Enter correct value.	16
		b. Accept current value by pressing **[ENTER]**.	16
16.	`Edit More Elements?`	a. Edit more elements by entering **Y**.	14
		b. Accept data by entering **N**.	17
17.		a. If you enter data from keyboard.	17
		b. Otherwise.	20
18.	`Record Data?`	a. Record data by entering **Y**.	19
		b. Continue program by entering **N**.	20
19.	`Enter Device.Filename:`	Enter the device number and filename on which to record data.	20
20.	`Input Sample Two...`	Displays prompt. Proceed by pressing **[ENTER]**.	21
21.		a. If you enter data from keyboard.	23
		b. Otherwise.	22
22.	`Enter Device.Filename:`	Enter the device number and filename on which data is stored.	24
23.	`Enter Y(###):`	a. Enter Xi.	24
		b. Indicate end of data.	24
24.	`Edit?`	a. Edit data by entering **Y**.	25
		b. Accept data by entering **N**.	30
25.	`Edit All Data?`	a. Edit all data by entering **Y**.	25
		b. Edit single elements by entering **N**.	26
26.	`Enter Y(###): ####`	a. Enter correct value.	28
		b. Accept current value by pressing **[ENTER]**.	28
27.	`Enter Element #:`	Enter the number of the element to be edited.	27
28.	`Enter Y(###): ####`	a. Enter correct value.	29
		b. Accept current value by pressing **[ENTER]**.	29

(continued)

(continued)

Step	Display	Procedure/Comment	Goto
29.	`Edit More Elements?`	a. Edit more elements by entering **Y**.	27
		b. Accept data by entering **N**.	30
30.		a. If you enter data from keyboard.	31
		b. Otherwise.	33
31.	`Record Data?`	a. Record data by entering **Y**.	32
		b. Continue program by entering **N**.	33
32.	`Enter Device.Filename:`	Enter the device number and filename on which to record data.	33
33.	`See Rank And Values?`	a. Display values and their rank based on merged data by entering **Y**.	34
		b. Continue program by entering **N**.	34
34.	`See X-Statistics?`	a. Display X-statistics by entering **Y**.	35
		b. Continue program by entering **N**.	38
35.	`Sum Of X-Ranks=`	Displays sum of X-ranks. Proceed by pressing **[ENTER]**.	36
36.	`W(X)=`	Displays Mann-Whitney X-statistic. Proceed by pressing **[ENTER]**.	37
37.	`Z(X)=`	Displays standard deviate for X-statistic. Proceed by pressing **[ENTER]**.	38
38.	`See Y-Statistics?`	a. Display Y-statistics by entering **Y**.	39
		b. Continue program by entering **N**.	42
39.	`Sum Of Y-Ranks=`	Displays sum of Y-ranks. Proceed by pressing **[ENTER]**.	40
40.	`W(Y)=`	Displays Mann-Whitney Y-statistic. Proceed by pressing **[ENTER]**.	41

(continued)

(continued)

Step	Display	Procedure/Comment	Goto
41.	`Z(Y)=`	Displays standard deviate for Y-statistic. Proceed by pressing **[ENTER]**.	42
42.	`Rank Mean=`	Displays the rank mean. Proceed by pressing **[ENTER]**.	43
43.	`Rank Variance=`	Displays the rank variance. Proceed by pressing **[ENTER]**.	44
44.	`Exit Program?`	a. Exit program by entering **Y**. b. Continue program by entering **N**.	STOP 4

Example

The Means and Moments example showed that the distribution of the heights of a group of men had the same shape as the distribution of the heights of their sons. The only difference detected was that the second distribution appeared to be shifted slightly to the right. The data used in the example is repeated below.

Fathers: 67.2, 65.0, 68.3, 69.9, 66.3, 69.7, 69.5, 72.9, 70.2, 74.1.
Sons: 68.4, 65.3, 66.5, 69.0, 73.6, 75.9, 69.7, 69.8, 71.0, 70.8, 67.7, 74.4, 69.9, 71.5, 71.1.

Test the hypotheses

H_0: the means of the distribution are equal, i.e., $f(x) = g(x)$

against

H_1: the means of the distributions differ by a positive constant c, such that $f(x + c) = g(x)$.

Perform this test at the 95% confidence level.

Select the Rank Sum Test program by entering **RUN "RSUM"**.

Step	Display	Procedure/Comment
1.	`RANK SUM TEST`	Program name.
2.	`Use Printer?`	Enter **N**.
3.	`Enter Data By Keyboard?`	Enter **Y**.
4.	`Enter Size For Sample 1: 0`	Enter **10**.
5.	`Enter Size For Sample 2: 0`	Enter **15**.
6.	`Input Sample One...`	Proceed by pressing **[ENTER]**.
7.	`Enter X(1): 0`	Enter **67.2**.
8.	`Enter X(2): 0`	Enter **65**.
9.	`Enter X(3): 0`	Enter **68.3**.
10.	`Enter X(4): 0`	Enter **69.9**.
11.	`Enter X(5): 0`	Enter **66.3**.
12.	`Enter X(6): 0`	Enter **69.7**.
13.	`Enter X(7): 0`	Enter **69.5**.
14.	`Enter X(8): 0`	Enter **72.9**.
15.	`Enter X(9): 0`	Enter **70.2**.
16.	`Enter X(10): 0`	Enter **74.1**.
17.	`Edit?`	Enter **N**.
18.	`Record Data?`	Enter **N**.
19.	`Input Sample Two...`	Proceed by pressing **[ENTER]**.
20.	`Enter Y(1): 0`	Enter **68.4**.
21.	`Enter Y(2): 0`	Enter **65.3**.
22.	`Enter Y(3): 0`	Enter **66.5**.
23.	`Enter Y(4): 0`	Enter **69**.
24.	`Enter Y(5): 0`	Enter **73.6**.

(continued)

(continued)

Step	Display	Procedure/Comment
25.	Enter Y(6): 0	Enter **75.9**.
26.	Enter Y(7): 0	Enter **69.7**.
27.	Enter Y(8): 0	Enter **69.8**.
28.	Enter Y(9): 0	Enter **71**.
29.	Enter Y(10): 0	Enter **70.8**.
30.	Enter Y(11): 0	Enter **67.7**.
31.	Enter Y(12): 0	Enter **77.4**.
32.	Enter Y(13): 0	Enter **69.9**.
33.	Enter Y(14): 0	Enter **71.5**.
34.	Enter Y(15): 0	Enter **72.1**.
35.	Edit?	Enter **Y**.
36.	Edit All Data?	Enter **N**.
37.	Enter Element #: 0	Enter **15**.
38.	Enter Y(15): 72.1	Enter **71.1**.
39.	Edit More Elements?	Enter **N**.
40.	Record Data?	Enter **N**.
41.	See Rank And Values?	Enter **Y**.
42.	X(1)= 65 Rank= 1 Y(1)= 65.3 Rank= 2 X(2)= 66.3 Rank= 3	Displays all requested values. Proceed after each output by pressing **[ENTER]**.

(continued)

(continued)

Step	Display	Procedure/Comment
	Y(2)= 66.5 Rank= 4 X(3)= 67.2 Rank= 5 Y(3)= 67.7 Rank= 6 X(4)= 68.3 Rank= 7 Y(4)= 68.4 Rank= 8 Y(5)= 69 Rank= 9 X(5)= 69.5 Rank= 10 X(6)= 69.7 Y(6)= 69.7 Rank= 11.5 Y(7)= 69.8 Rank= 13 X(7)= 69.9 Y(8)= 69.9 Rank= 14.5 X(8)= 70.2 Rank= 16 Y(9)= 70.8 Rank= 17	

(continued)

(continued)

Step	Display	Procedure/Comment
	Y(10)= 71 Rank= 18 Y(11)= 71.1 Rank= 19 Y(12)= 71.5 Rank= 20 X(9)= 72.9 Rank= 21 Y(13)= 73.6 Rank= 22 X(10)= 74.1 Rank= 23 Y(14)= 75.9 Rank= 24 Y(15)= 77.4 Rank= 25	
43.	See X-Statistics?	Enter **Y**.
44.	Sum Of X-Ranks= 112 W(X)= 93 Z(X)= .9984603532	Displays all requested values. Proceed after each ouput by pressing **[ENTER]**.
45.	See Y-Statistics?	Enter **Y**.
46.	Sum Of Y-Ranks= 213 W(Y)= 57 Z(Y)=-.9984603532 Rank Mean= 75 Rank Variance= 325	Displays all requested values. Proceed after each output by pressing **[ENTER]**.
47.	Exit Program?	Enter **Y**.

Analysis of Variance

When trying to compare the means of various sets of data with different variances, you can use Analysis of Variance techniques to test the hypothesis that a number of populations have the same mean. The test is made by using the sample means to estimate the variance of the population. This estimated variance is then compared to another estimate of the population variance made from the differences between individual elements of the same sample. The F-distribution is used in the comparison of the two variances and indicates whether to accept or reject the null hypothesis.

Two assumptions concerning the analysis of variance are that the populations are normally distributed and that the population variances are approximately equal.

One-Way ANOVA—"AOV1"

The One-Way ANOVA test is used to analyze several sample populations where only one factor varies. Sample populations used in one-way analysis of variance are often called treatment groups. Each treatment group i ($i = 1, 2, \ldots, K$) consists of n_i observations. Each observation in the treatment group i is denoted as x_{ij} ($j = 1, 2, \ldots, n_i$). The different groups need not have the same number of observations.

In the following, let

$$N = \sum_{i=1}^{K} n_i$$

where N is the total number of observations. With a 6K byte memory capacity computer, you may enter up to 200 elements. The fundamental variance formula associated with the F test's hypothesis is

$$s^2 = \sum_{i=1}^{K} \frac{(x_i - \bar{x})^2}{N - 1}$$

This equation can be modified to the formula

$$s^2 = \frac{\sum_{i=1}^{K} x_i^2 - \left(\sum_{i=1}^{K} x\right)^2 /N}{N-1}$$

It is convenient to use the above formula to find variances since the necessity of finding means and deviations from the means is eliminated.

The following notation is used.

T—Specifies a total.

•—Subscript dot specifies composition of the sum.

Therefore, $T_{i\bullet}$ indicates the total of the ith row, and the dot indicates that the total includes all column values in that row. $T_{\bullet\bullet}$ indicates the grand total of all the observations. The following table shows a standard ANOVA chart using this notation.

	Replicates										
Level	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**$T_{i\bullet}$**
1	2	6	4	13	5	8	4	6			48
2	6	4	4	1	8	2	12	1	5	2	45
3	2	1	3	3	1	7	1	4	2		24
										$T_{\bullet\bullet}=$	117

The total sum of squares (SSΣ) is the numerator in the revised equation for estimating the variance. The formula for finding the total sum of squares is

$$SS\Sigma = \sum_{i=1}^{K} \sum_{j=1}^{n_i} x_{ij}^2 - (T_{\bullet\bullet})^2/N$$

where subscript i is the row number and subscript j is the column number. The row totals ($T_{i\bullet}$) are used to calculate sum of squares for treatment rows (SST) by the following formula.

$$SST = \sum_{i=1}^{K} T_{i\bullet}^2/n_i - T_{\bullet\bullet}^2/N)$$

Finally, the sum of squares for error (SSE) may be calculated from the following formula.

$SSE = SS\Sigma - SST$

Now, by dividing SST and SSE by their respective degrees of freedom (df), we have the variance terms for the F test.

$df\Sigma = N - 1$

$dfT = K - 1$

$dfE = N - K$

Given the above terms and the preceding table, the ANOVA table for those values can be constructed. The mean square is the total sum of squares divided by df. The two mean square formulas used in this program are provided below and are followed by the ANOVA table constructed from the values in the preceding standard ANOVA table.

$MST = SST/dfT$

$MSE = SSE/dfE$

Source	**Sum of Squares**	**df**	**Mean Square**
Rows	47.5	2	23.75 (MSR)
Error	216.5	24	9.02 (MSE)
Total	264	26	

The F-statistic

$F = MST/MSE$

with dfT in the numerator and dfE in the denominator is then calculated by the program. You can use this data and the F-distribution program found in the "Theoretical Distributions" section to test the hypothesis that the means of your populations are equal.

Reference: *Statistics*, Rickmers & Todd, McGraw-Hill, 1967, pp. 162-166.

User Instructions—One-Way ANOVA

Select the One-Way ANOVA program by entering **RUN "AOV1"**.

Step	Display	Procedure/Comment	Goto
1.	ONE WAY ANOVA	Program name.	2
2.	Use Printer?	a. Accept printer usage by entering **Y**.	3
		b. Reject printer usage by entering **N**.	4
3.	Enter Filename:	Enter output device name.	4
4.	Enter Data By Keyboard?	a. Enter data from keyboard by entering **Y**.	5
		b. Enter data from peripheral device by entering **N**.	5
5.	Enter Treatment Number:	Enter the number of the treatment you want to enter.	6
6.	Enter No. Of Elements:	Enter the number of replicates in your sample.	7
7.		a. If you enter data from keyboard.	9
		b. Otherwise.	8
8.	Enter Device.Filename:	Enter the device number and filename on which data is stored.	11
9.	To End Input Enter 'E'	Displays prompt. Proceed by pressing **[ENTER]**.	10
10.	Enter X(##,##):	Enter Xij.	11
11.	Edit?	a. Edit data by entering **Y**.	12
		b. Accept data by entering **N**.	17
12.	Edit All Data?	a. Edit all data by entering **Y**.	13
		b. Edit single elements by entering **N**.	14
13.	Enter X(##,##): ####	Enter correct value.	17
14.	Enter Column Number:	Enter the number of the row you want to edit.	15
15.	Enter X(##,##): ####	a. Enter correct value.	16
		b. Accept current value by pressing **[ENTER]**.	16

(continued)

(continued)

Step	Display	Procedure/Comment	Goto
16.	Edit More Elements?	a. Edit more values by entering **Y**.	14
		b. Accept data by entering **N**.	17
17.		a. If you enter data from keyboard.	18
		b. Otherwise.	20
18.	Record Data?	a. Record data by entering **Y**.	21
		b. Continue program by entering **N**.	20
19.	Enter Device.Filename:	Enter the device number and filename on which to record data.	20
20.	More Treatments?	a. Enter more treatments by entering **Y**.	5
		b. Continue program by entering **N**.	21
21.	SS(TOTAL)=	Displays total sum of squares. Proceed by pressing **[ENTER]**.	22
22.	df(TOTAL)=	Displays total degrees of freedom. Proceed by pressing **[ENTER]**.	23
23.	SST=	Displays sum of squares treatment. Proceed by pressing **[ENTER]**.	24
24.	dfT=	Displays degrees of freedom treatment. Proceed by pressing **[ENTER]**.	28
25.	MST=	Displays mean square treatment. Proceed by pressing **[ENTER]**.	26
26.	SSE=	Displays sum of squares error. Proceed by pressing **[ENTER]**.	27
27.	dfE=	Displays degrees of freedom error. Proceed by pressing **[ENTER]**.	28
28.	MSE=	Displays mean square error. Proceed by pressing **[ENTER]**.	29

(continued)

(continued)

Step	Display	Procedure/Comment	
29.	F=	Displays F statistic. Proceed by pressing **[ENTER]**.	30
30.	Exit Program?	a. Exit program by entering **Y**.	STOP
		b. Continue program by entering **N**.	4

Example

By extending the example from the "Unpaired t-test" section, we can compare the effects of three drugs used for the relief of pain. Assume that three groups of patients with the same illness are chosen for the experiment and are given drugs x, y, and z respectively. The resulting number of hours of relief for each patient is given below.

Drug x: 2, 6, 4, 13, 5, 8, 4, 6
Drug y: 6, 4, 4, 1, 8, 2, 12, 1, 5, 2
Drug z: 2, 1, 3, 3, 1, 7, 1, 4, 2

Test the hypotheses

H_0: All of the drugs relieve pain for the same amount of time ($\bar{x} = \bar{y} = \bar{z}$)

against

H_1: All of the drugs do not relieve pain for the same amount of time.

Since the t-test can only be used for comparing two sample populations, we use the one-way analysis of variance to perform this test.

Select the One-Way ANOVA program by entering **RUN "AOV1"**.

Step	Display	Procedure/Comment
1.	ONE WAY ANOVA	Program name.
2.	Use Printer?	Enter **N**.
3.	Enter Data By Keyboard?	Enter **Y**.
4.	Enter Treatment Number: 1	Accept current value by pressing **[ENTER]**.
5.	Enter No. Of Elements: 0	Enter **8**.
6.	To End Input Enter 'E'	Proceed by pressing **[ENTER]**.
7.	Enter X(1,1):	Enter **2**.
8.	Enter X(1,2):	Enter **6**.
9.	Enter X(1,3):	Enter **4**.
10.	Enter X(1,4):	Enter **13**.
11.	Enter X(1,5):	Enter **5**.
12.	Enter X(1,6):	Enter **8**.
13.	Enter X(1,7):	Enter **48**.
14.	Enter X(1,8):	Enter **6**.
15.	Edit?	Enter **Y**.
16.	Edit All Data?	Enter **N**.
17.	Enter Column Number:	Enter **7**.
18.	Enter X(1,7): 48	Enter **4**.
19.	Edit More Elements?	Enter **N**.
20.	Record Data?	Enter **N**.
21.	More Treatments?	Enter **Y**.
22.	Enter Treatment Number: 2	Accept current value by pressing **[ENTER]**.
23.	Enter No. Of Elements: 8	Enter **10**.
24.	To End Input Enter 'E'	Proceed by pressing **[ENTER]**.

(continued)

(continued)

Step	Display	Procedure/Comment
25.	Enter X(2,1): 0	Enter **6**.
26.	Enter X(2,2): 0	Enter **4**.
27.	Enter X(2,3): 0	Enter **4**.
28.	Enter X(2,4): 0	Enter **1**.
29.	Enter X(2,5): 0	Enter **8**.
30.	Enter X(2,6): 0	Enter **2**.
31.	Enter X(2,7): 0	Enter **12**.
32.	Enter X(2,8): 0	Enter **1**.
33.	Enter X(2,9): 0	Enter **5**.
34.	Enter X(2,10): 0	Enter **2**.
35.	Edit?	Enter **N**.
36.	Record Data?	Enter **N**.
37.	More Treatments?	Enter **Y**.
38.	Enter Treatment Number: 3	Accept current value by pressing **[ENTER]**.
39.	Enter No. Of Elements: 10	Enter **9**.
40.	To End Input Enter 'E'	Proceed by pressing **[ENTER]**.
41.	Enter X(3,1): 0	Enter **2**.
42.	Enter X(3,2): 0	Enter **1**.
43.	Enter X(3,3): 0	Enter **3**.
44.	Enter X(3,4): 0	Enter **3**.
45.	Enter X(3,5): 0	Enter **1**.
46.	Enter X(3,6): 0	Enter **7**.
47.	Enter X(3,7): 0	Enter **1**.
48.	Enter X(3,8): 0	Enter **4**.

(continued)

(continued)

Step	Display	Procedure/Comment
49.	Enter X(3,9): 0	Enter **2**.
50.	Edit?	Enter **N**.
51.	Record Data?	Enter **N**.
52.	More Treatments?	Enter **N**.
53.	SS(TOTAL)= 264 df(TOTAL)= 26 SST= 47.5 dfT= 2 MST= 23.75 SSE= 216.5 dfE= 24 MSE= 9.020833333 F= 2.632794457	Displays all requested values. Proceed after each output by pressing **[ENTER]**.
54.	Exit Program?	Enter **Y**.

Two-Way ANOVA—"AOV2"

There are many cases where the One-Way ANOVA program is inappropriate. Such situations involve two or more factors and a possible influence of one factor upon another. When the influence is dependent, interaction is said to occur. The requirement for a two-factor situation is that replication (or repetition) of test procedures must be made so that some estimate of random error can be made. Note that replication requires the preparation of two or more independent samples, and these samples must be treated as similarly as possible. Duplication, unlike replication, is simply making two or more measurements on a given sample which has come from a single experimental procedure.

Notation in the Two-Way ANOVA program closely follows that of the One-Way ANOVA program.

i – row
j – column
k – replicate

For example, $T_{\bullet j \bullet}$ indicates a column total for all row and replicate numbers. The following table, utilizing the above notation, shows a standard two factor experiment replicated twice. The experiment consists of mixing film ingredients for a certain time at a certain temperature. The results attained in the experiment are a percentage change of processing time from the normal processing time.

		Temperature	
Time	50	70	$T_{i\bullet\bullet}$
1 hour	10	−20	
	30	−40	
	$T_{1j\bullet}$ 40	−60	−20
2 hours	0	50	
	40	40	
	$T_{2j\bullet}$ 40	90	130
$T_{\bullet j \bullet}$	80	30	$110 = T_{\bullet\bullet\bullet}$

The total sum of squares (SSΣ) is found by

$$SS\Sigma = \sum_{h=1}^{K} \sum_{j=1}^{c} \sum_{i=1}^{r} x_{hij}^2 - (T_{\bullet\bullet\bullet})^2/N$$

where N = rows × columns × replicates = total observations (with a 6K byte memory capacity computer, this total may not exceed 400 elements). The sum of squares for rows is calculated by the following formula.

$$SSR = \frac{1}{ck} \sum_{i=1}^{r} T_{i\bullet\bullet}^2 - (T_{\bullet\bullet\bullet})^2/N$$

where ck = columns × replicates.

Similarly, the sum of squares for the columns is

$$SSC = \frac{1}{rk} \sum_{i=1}^{c} T_{\bullet i\bullet}^2 - (T_{\bullet\bullet\bullet})^2/N$$

where rk = rows × replicates.

Finally, the sums of squares for interaction is found by the formula

$$SSI = \frac{1}{k} \sum_{j=1}^{c} \sum_{i=1}^{r} T_{ij\bullet}^2 - SSR - SSC - (T_{\bullet\bullet\bullet})^2/N$$

The sum of squares for error is arrived at by subtraction of the above sums of squares from the total sum of squares.

$SS\Sigma = SS\Sigma - SSR - SSC - SSI$

The degrees of freedom (df) for each respective formula above are

$df = N - 1$
$dfR = r - 1$
$dfC = c - 1$
$dfI = (r - 1)(c - 1)$
$df = df\Sigma - dfR - dfC - dfI$
$MSR = SSR/dfR$
$MSC = SSC/dfC$
$MSI = SSI/dfI$
$MSE = SSE/dfE$

Utilizing the data previously provided in this section, the following ANOVA table may be constructed.

Source	Sum of Squares	df	Mean Square	F Ratio
Time	2,812.5	1	2,812.5	9
Temperature	312.5	1	312.5	1
Interaction	2,812.5	1	2,812.5	9
Error	1,250.0	4	312.5	
Total	7,187.5	7		

As in the One-Way ANOVA table, the mean square terms are the sum of squares divided by their respective degrees of freedom. The F ratios are computed by dividing each source mean square by the error mean square as indicated by the following formulas.

$$F_r = \frac{MSR}{MSE}$$

$$F_c = \frac{MSC}{MSE}$$

$$F_i = \frac{MSI}{MSE}$$

User Instructions—Two-Way ANOVA

Select the Two-Way ANOVA program by entering **RUN "AOV2"**.

Step	Display	Procedure/Comment	Goto
1.	`TWO WAY ANOVA`	Program name.	2
2.	`Use Printer?`	a. Accept printer usage by entering **Y**.	3
		b. Reject printer usage by entering **N**.	4
3.	`Enter Filename:`	Enter output device name.	4
4.	`Enter Data By Keyboard?`	a. Enter data from keyboard by entering **Y**.	5
		b. Enter data from peripheral device by entering **N**.	5
5.	`Enter No. Of Rows:`	Enter the number of rows in your sample.	6
6.	`Enter No. Of Columns:`	Enter the number of columns in your sample.	7
7.	`Enter No. Of Replicates:`	Enter the number of replicates in your sample.	8
8.		a. If you enter data from keyboard.	9
		b. Otherwise.	8
9.	`Enter Device.Filename:`	Enter the device number and filename on which data is stored.	10
10.	`Enter (##,##,##):`	Enter Xijk.	11
11.	`Edit?`	a. Edit data by entering **Y**.	12
		b. Accept data by entering **N**.	19
12.	`Edit All Data?`	a. Edit all data by entering **Y**.	13
		b. Edit single elements by entering **N**.	14
13.	`Enter (##,##,##): ####`	a. Enter correct value.	13
		b. Accept current value by pressing **[ENTER]**.	13
14.	`Enter Row Number:`	Enter the number of the row you want to edit.	15
15.	`Enter Column Number:`	Enter the number of the column you want to edit.	16

(continued)

(continued)

Step	Display	Procedure/Comment	Goto
16.	`Enter Replicate Number:`	Enter the number of the replicate you want to edit.	17
17.	`Enter (##,##,##): ####`	a. Enter correct value.	18
		b. Accept current value by pressing **[ENTER]**.	18
18.	`Edit More Elements?`	a. Edit more values by entering **Y**.	14
		b. Accept data by entering **N**.	19
19.		a. If you enter data from keyboard.	20
		b. Otherwise.	22
20.	`Record Data?`	a. Record data by entering **Y**.	18
		b. Continue program by entering **N**.	19
21.	`Enter Device.Filename:`	Enter the device number and filename on which to record data.	19
22.	`Display Anova Table?`	a. Display ANOVA table by entering **Y**.	23
		b. Continue program by entering **N**.	??
23.	`SS(TOTAL)=`	Displays total sum of squares. Proceed by pressing **[ENTER]**.	24
24.	`df(TOTAL)=`	Displays total degrees of freedom. Proceed by pressing **[ENTER]**.	25
25.	`SSR=`	Displays sum of squares for rows. Proceed by pressing **[ENTER]**.	26
26.	`dfR=`	Displays degrees of freedom for rows. Proceed by pressing **[ENTER]**.	27
27.	`MSR=`	Displays mean square for rows. Proceed by pressing **[ENTER]**.	28
28.	`SSC=`	Displays sum of squares for columns. Proceed by pressing **[ENTER]**.	29

(continued)

(continued)

Step	Display	Procedure/Comment	Goto
29.	`dfC=`	Displays degrees of freedom for columns. Proceed by pressing **[ENTER]**.	30
30.	`MSC=`	Displays mean square for columns. Proceed by pressing **[ENTER]**.	31
31.	`SSI=`	Displays sum of squares interaction. Proceed by pressing **[ENTER]**.	32
32.	`dfI=`	Displays degrees of freedom for interaction. Proceed by pressing **[ENTER]**.	33
33.	`MSI=`	Displays mean square for interaction. Proceed by pressing **[ENTER]**.	34
34.	`SSE=`	Displays sum of squares error. Proceed by pressing **[ENTER]**.	35
35.	`dfE=`	Displays degrees of freedom error. Proceed by pressing **[ENTER]**.	36
36.	`MSE=`	Displays mean squares error. Proceed by pressing **[ENTER]**.	37
37.		a. If sample contains only one row or if SSI and SSE have been pooled.	39
		b. Otherwise.	38
38.	`Fi=`	Displays F ratio for interaction. Proceed by pressing **[ENTER]**.	39
39.	`Fr=`	Displays F ratio for rows. Proceed by pressing **[ENTER]**.	40
40.	`Fc=`	Displays F ratio for columns. Proceed by pressing **[ENTER]**.	41

(continued)

(continued)

Step	Display	Procedure/Comment	Goto
41.		a. If interaction terms have been pooled.	43
		b. Otherwise.	42
42.	Pool SSI And SSE?	a. Pool interaction terms with error terms by entering **Y**.	22
		b. Continue program by entering **N**.	43
43.	Exit Program?	a. Exit program by entering **Y**.	STOP
		b. Continue program by entering **N**.	4

Example

In the manufacture of a photographic film emulsion, it is customary to mix the ingredients and to allow the mixture to set for a certain time at a certain temperature. You wish to know whether there is a significant interaction between time and temperature for a particular set of ingredients. Since the problem involves two factors (time and temperature) plus a possible interaction, the experiment is replicated to obtain a separate estimate of error. The results are expressed as a percentage change from the normal processing time of the film. The table is shown below.

	Temperature		
Time	**60**	**70**	$T_{i\bullet\bullet}$
1 hour	15	−15	
	25	−30	
	T_{1j} 40	−45	−5
2 hours	5	55	
	45	45	
	T_{2j} 50	90	140
$T_{\bullet j\bullet}$	90	45	$135 = T_{\bullet\bullet\bullet}$

Test the following three null hypotheses at the 95% confidence level.

1. The changes in time of the holding mixtures produce no effect.
2. The changes in temperature produce no effect.
3. There is no interaction of time and temperature.

Select the Two-Way ANOVA program by entering **RUN "AOV2"**.

Step	Display	Procedure/Comment
1.	TWO WAY ANOVA	Program name.
2.	Use Printer?	Enter **N**.
3.	Enter Data By Keyboard?	Enter **Y**.
4.	Enter No. Of Rows: 0	Enter **2**.
5.	Enter No. Of Columns: 0	Enter **2**.
6.	Enter No. Of Replicates: 0	Enter **2**.
7.	Enter (1,1,1): 0	Enter **15**.
8.	Enter (1,1,2): 0	Enter **25**.
9.	Enter (1,2,1): 0	Enter **−15**.
10.	Enter (1,2,2): 0	Enter **−30**.
11.	Enter (2,1,1): 0	Enter **55**.
12.	Enter (2,1,2): 0	Enter **45**.
13.	Enter (2,2,1): 0	Enter **55**.
14.	Enter (2,2,2): 0	Enter **45**.
15.	Edit?	Enter **Y**.
16.	Edit All Data?	Enter **N**.
17.	Enter Row Number: 0	Enter **2**.
18.	Enter Column Number: 0	Enter **1**.
19.	Enter Replicate Number: 0	Enter **1**.

(continued)

(continued)

Step	Display	Procedure/Comment
20.	Enter (2,1,1): 55	Enter **5**.
21.	Edit More Elements?	Enter **N**.
22.	Record Data?	Enter **N**.
23.	Display Anova Table?	Enter **Y**.
24.	SS(TOTAL)= 6446.875 df(TOTAL)= 7 SSR= 3003.125 dfR= 1 MSR= 3003.125 SSC= 153.125 dfC= 1 MSC= 153.125 SSI= 2278.125 dfI= 1 MSI= 2278.125 SSE= 1012.5 dfE= 4 MSE= 253.125 Fi= 9 Fr= 11.86419753 Fc= .6049382716	Displays all requested values. Proceed after each output by pressing **[ENTER]**.
25.	Pool SSI And SSE?	Enter **Y**.
26.	Display Anova Table?	Enter **Y**.

(continued)

(continued)

Step	Display	Procedure/Comment
27.	SS(TOTAL)= 6446.875 df(TOTAL)= 7 SSR= 3003.125 dfR= 1 MSR= 3003.125 SSC= 153.125 dfC= 1 MSC= 153.125 SSE= 3290.625 dfE= 5 MSE= 658.125 Fr= 4.563152896 Fc= .232668566	Displays all requested values. Proceed after each output by pressing **[ENTER]**.
28.	Exit Program?	Enter **Y**.

Theoretical Distributions

A statistical experiment leads to the evaluation of a test statistic. A test statistic is a random variable with a known frequency of occurrence or distribution. Since you know the distribution of this statistic, you can determine the range of values that it may take when your null hypothesis is true. This range is called the acceptance region. If your value falls outside of the acceptance region, it is in what is known as the rejection or critical region. The sizes of these regions depend upon the confidence level at which you wish to make your test.

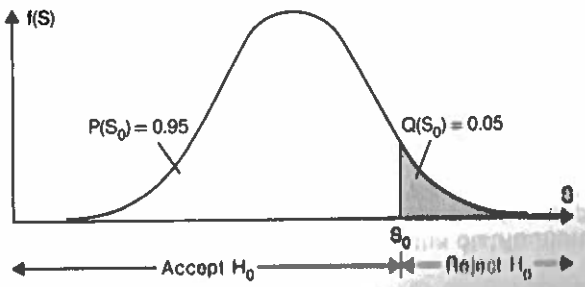
There is always the possibility of chance variations occurring when you sample a population. Confidence levels are used to take these chance variations into consideration to protect against rejecting your null hypothesis (H_0) when it is actually true. For example, if you are testing at the 95% confidence level and the test statistic falls inside the rejection region, you are 95% sure that your null hypothesis is incorrect and not caused by chance variations in sampling.

Testing at the 95% confidence level is also known as testing at the 5% level of significance. In other words, you mistakenly reject H_0 in 5% of your experiments.

For example, the hypotheses tested in the upper-tailed test case are

H_0: $S \leq S_0$ against H_1: $S > S_0$.

In the above, S is the test statistic and S_0 is the critical point or dividing line between the acceptance and rejection regions. If you were testing at the 5% significance level, your distribution curve might look like the following illustration.



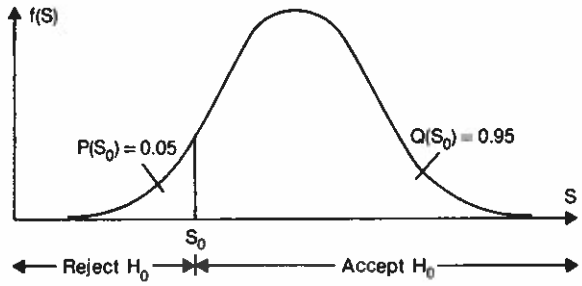
The shaded area, designated $Q(S_0)$, is the upper-tail area of the distribution curve. Since you are testing at the 5% significance level, S_0 has been chosen such that the upper-tail area equals 0.05 of the curve. If your test statistic is less than S_0, you will fail to reject the null hypothesis. The rejection region for this test is $(S_0, \cong)$.

To determine if your statistic falls within the rejection region, calculate Q(S) using an appropriate program from this section. If Q(S) is less than 0.05, you reject H_0 while using a 95% confidence level. This is the same as rejecting H_0 when P(S) is greater than 0.95.

In the case of a lower-tailed test, the hypotheses being tested are

H_0: $S \geq S_0$ against H_1: $S < S_0$.

The lower-tailed area of a frequency distribution curve is designated by P(S) in the following graph.

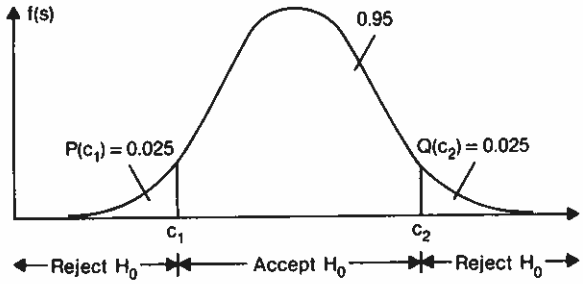


The rejection region for this test is $(-\cong, S_0)$. To complete this test, calculate P(S) for your statistic and reject H_0 if this value is less than 0.05. Alternatively, you may determine Q(S) and reject H_0 if this value exceeds 0.95.

The hypotheses of a two-tailed test are

H_0: $S = S_0$ against H_1: $S \neq S_0$.

For this type of test, the critical region is divided into two equal parts as shown below. Note that testing at the 5% significance level causes each tail area to be set at 0.025.



In this case, you have two critical points, c_1 and c_2. These points establish what is known as a confidence interval for S_0. Specifically, (c_1, c_2) is a 95% confidence interval for S_0. If your statistic is such that $c_1 \leq S \leq c_2$, you would fail to reject the null hypothesis. To determine whether S is in this range or not, simply calculate P(S) or Q(S) and check to see if $0.025 \leq P(S) \leq 0.975$ or $0.025 \leq Q(S) \leq 0.975$. When one of these conditions is met, the other is also true.

Selecting a Confidence Level

When you select a confidence level for your experiment, it is important that you realize how the statistical process works. Keep in mind that the amount of information you have in your sample does not change. Therefore, if you select a high level of confidence, your answer may vary greatly.

For example, suppose that a mechanic looks at your car and tells you he is pretty sure it will cost between $80 and $100 to fix it. However, if you tell him that he has to be 99.9% sure of his estimate, he will probably estimate a wider range of $50 to $200. If your experiment requires more confidence without a larger variance in your results, you may need to take a larger sample.

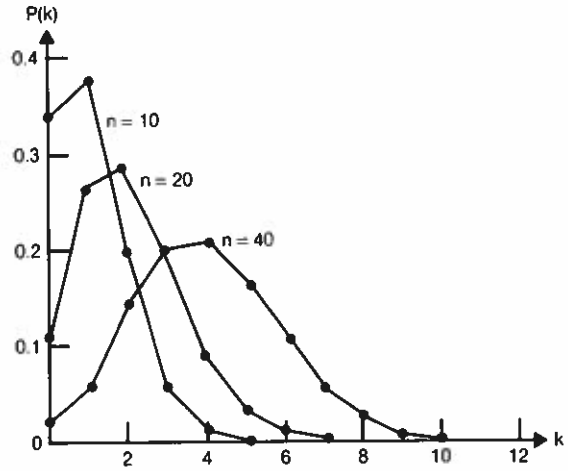
Overall, you can increase your protection against rejecting a true H_0 by testing at a lower level of significance. However, as the level of significance decreases, the risk of failing to reject a false H_0 increases. If failing to reject a false H_0 is costly, you can gain protection against this type of error by increasing the sample size.

Binomial Distribution—"BINOM"

The Binomial Distribution program may be used when your variables comply with the following conditions.

- The experiment consists of a fixed number of statistically independent trials.
- Each trial results in either success or failure.
- Each trial has a constant probability of success (p) and failure (1 – p).

Generally, these conditions are upheld when your sample is obtained from a finite population with replacement or from an infinite population. The drawing below illustrates the binomial distribution, where n is the number of trials, k is the number of successes in the experiment, and the probability of success is 0.1.



As you can see, the normal curve is frequently a good approximation for the binomial distribution if both np and $n(1 - p) \geq 5$.

You can use this program to calculate the following probabilities for n trials of an experiment.

Probability of exactly k successes $= f(k;n,p) = f(k)$.

Probability of k or fewer successes $= F(k;n,p) = P(k)$.

Probability of more than k successes $= 1 - F(k;n,p) = Q(k)$.

This program also calculates the mean and standard deviation of a binomially distributed population.

Method Used—Binominal Distribution Program

The probability function of a binomial distribution is given by

$$f(k; n, p) = \begin{cases} \binom{n}{k} p^k (1-p)^{n-k}, & k = 0, 1, 2, \ldots, n; \\ 0, & \text{elsewhere} \end{cases}$$

The cumulative distribution function of a random variable with binomial distribution is

$$F(k; n, p) = \sum_{j=0}^{k} f(j; n, p).$$

The mean and standard deviation are determined by

$$\bar{x} = np$$

and

$$\sigma = np(1-p)$$

User Instructions—Binomial Distribution

Select the Binomial Distribution program by entering **RUN "BINOM"**.

Step	Display	Procedure/Comment	Goto
1.	BINOMIAL DISTRIBUTION	Program name.	2
2.	Use Printer?	a. Accept printer usage by entering **Y**. b. Reject printer usage by entering **N**.	3 4
3.	Enter Filename:	Enter output device name.	4
4.	Enter n:	Enter the number of trials.	5
5.	Enter p:	Enter the probability of success.	6
6.	Enter k:	Enter the number of successes.	7
7.	Mean=	Displays the mean. Proceed by pressing **[ENTER]**.	8
8.	Standard Dev=	Displays the standard deviation. Proceed by pressing **[ENTER]**.	9
9.	f(k)=	Displays the probability of exactly k successes. Proceed by pressing **[ENTER]**.	10
10.	P(k)=	Displays the probability of k or fewer successes. Proceed by pressing **[ENTER]**.	11
11.	Q(k)=	Displays the probability of more than k successes. Proceed by pressing **[ENTER]**.	12
12.	Exit Program?	a. Exit program by entering **Y**. b. Continue program by entering **N**.	STOP 4

Example 1

Suppose that you tossed a coin 50 times and obtained 28 heads. Test the hypotheses

H_0: the coin is balanced ($p = 0.5$)

against

H_1: the coin is balanced ($p \neq 0.5$)

at the 95% confidence level.

Select the Binomial Distribution program by entering **RUN "BINOM"**.

Step	Display	Procedure/Comment
1.	BINOMIAL DISTRIBUTION	Program name.
2.	Use Printer?	Enter **N**.
3.	Enter n: 0	Enter **50**.
4.	Enter p: 0	Enter **.5**.
5.	Enter k: 0	Enter **28**.
6.	Mean= 25 Standard Dev= 3.535533906 f(28)= .0788256707 P(28)= .8388818398 Q(28)= .1611181602	Displays all requested values. Proceed after each output by pressing **[ENTER]**.
7.	Exit Program?	Enter **Y**.

A two-tailed test is required for this exercise. You fail to reject H_0 if $0.025 \leq P(k) \leq 0.975$. Since the value above falls in this range, you have found no evidence that the coin is unbalanced.

Example 2

Find the probability of rolling exactly 2 fours in 5 rolls of a balanced die.

Select the Binomial Distribution program by entering **RUN "BINOM"**.

Step	Display	Procedure/Comment
1.	BINOMIAL DISTRIBUTION	Program name.
2.	Use Printer?	Enter **N**.
3.	Enter n: 0	Enter **5**.
4.	Enter p: 0	Enter **1/6**.
5.	Enter k: 0	Enter **2**.
6.	Mean= .8333333333 Standard Dev= .8333333333 f(2)= .1607510288 P(2)= .9645061728 Q(2)= .0354938272	Displays all requested values. Proceed after each output by pressing **[ENTER]**.
7.	Exit Program?	Enter **Y**.

There is a 16.1% chance of rolling exactly two fours in five rolls of a balanced die.

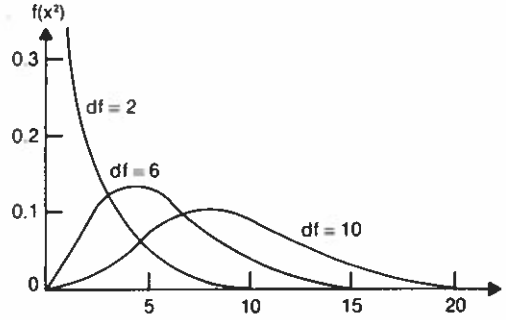
Chi-Square Distribution—"CHI2"

The chi-square distribution is the distribution of the variances of a normally distributed random variable. The chi-square statistic, denoted by χ^2, is most often used to establish confidence intervals for the standard deviation of a population since its distribution depends on the deviations of a sample. The following formula defines chi-square.

$$\chi^2 = \sum \frac{(f_e - f_a)^2}{f_e}$$

where f_a is the actual value of a given cell and f_e is the expected value of that cell.

The shape of a chi-square curve is controlled by the number of degrees of freedom (df) it has. As you can see below, the mean of the chi-square distribution is df. Also, the variance of each curve is equal to 2df. The range of a chi-square distribution is from zero to infinity since its distribution is defined as the sum of squared values.



When n is greater than 30, it is recommended that the following formula be used as an approximation of the standard unit normal distribution.

$$z = \sqrt{2\chi^2} - \sqrt{2df}$$

If this normal approximation is not used for n > 30, the inequality

$$e^{x^2/2} \leq 9.999999 \times 10^{99}$$

must be satisfied or erroneous results will be obtained with this program.

Method Used—Chi-Square Program

A series expansion is used to approximate $Q(\chi^2)$. When the degrees of freedom are odd and

$$\chi = \sqrt{\chi^2}$$

$$Q(\chi^2:df) = 2Q(\chi) + 2F(\chi) \sum_{r=1}^{\frac{df-1}{2}} \frac{\chi^{2r-1}}{1 \times 3 \times 5 \ldots (2r-1)}$$

where $Q(\chi)$ and $F(\chi)$ are computed using the Normal Distribution program. However, when the degrees of freedom are even,

$$Q(\chi^2:df) = e^{-m} \sum_{j=0}^{c-1} \frac{m^j}{j!}$$

where

$c = df/2$

and

$m = \chi^2/2$

You may then calculate the following probability manually.

$$P(\chi^2) = 1 - Q(\chi^2) = Pr(X \leq \chi^2)$$

Reference: *Handbook of Mathematical Functions*, Abramowitz and Stegun, National Bureau of Standards, 1964, p. 941.

User Instructions—Chi-Square Distribution

Select the Chi-Square Distribution program by entering **RUN "CHI2"**.

Step	Display	Procedure/Comment	Goto
1.	CHI-SQUARE DISTRIBUTION	Program name.	2
2.	Use Printer?	a. Accept printer usage by entering **Y**.	3
		b. Reject printer usage by entering **N**.	4
3.	Enter Filename:	Enter output device name.	4
4.	Enter df:	Enter the degrees of freedom.	5
5.	Enter Chi-Square:	Enter the chi-square value.	6
6.	Q(###)=	Displays the area of the curve to the right of the chi-square value. Proceed by pressing **[ENTER]**.	7
7.	Exit Program?	a. Exit program by entering **Y**.	STOP
		b. Continue program by entering **N**.	4

Example 1

If s^2 is the sample variance of a sample of size n selected from normal population, the distribution of s^2 can be obtained from the formula

$$s^2 = \chi^2(\sigma^2/n)$$

where chi-square is the chi-square statistic with n – 1 degrees of freedom and χ^2 is the population variance. This is equivalent to

$$\chi^2 = (ns^2/\sigma^2)$$

Now suppose that 20 scales of a certain make are tested for accuracy and yield a sample variance of 3 ounces. Use the above formula to verify that this sample variance does not contradict the assumption that the population variance is less than 2.5 ounces at the 95% confidence level.

Select the Chi-Square Distribution program by entering **RUN "CHI2"**.

Step	Display	Procedure/Comment
1.	CHI-SQUARE DISTRIBUTION	Program name.
2.	Use Printer?	Enter **N**.
3.	Enter df: 0	Enter **19**.
4.	Enter Chi-Square: 0	Enter **20*3/2.5**.
5.	Q(24/19)= .1961523586	Displays the area of the curve to the right of the chi-square value. Proceed by pressing **[ENTER]**.
6.	Exit Program?	Enter **Y**.

The actual hypotheses tested in this example are

$H_0: \sigma^2 \leq 2.5$ against $H_1: \sigma^2 > 2.5$.

However, to make this evaluation, you must test

$H_0: \chi^2 \leq ns^2/\sigma^2$ against $H_1: \chi^2 > ns^2/\sigma^2$

where χ^2 is chi-square with n – 1 degrees of freedom.

Due to the nature of the hypotheses, an upper-tailed test is required to complete this evaluation. If $Q(\chi^2) \leq 0.05$ (as defined by the 95% confidence level), you must reject H_0. In this case, $Q(\chi^2) = 0.1961524$, and you fail to reject H_0.

Example 2

A set of 10 data points yields a chi-square statistic of 3.129. Using a two-tailed test, does your hypothesis hold true at the 95% confidence level?

Select the Chi-Square Distribution program by entering **RUN "CHI2"**.

Step	Display	Procedure/Comment
1.	`CHI-SQUARE DISTRIBUTION`	Program name.
2.	`Use Printer?`	Enter **N**.
3.	`Enter df: 0`	Enter **9**.
4.	`Enter Chi-Square: 0`	Enter **3.129**.
5.	`Q(3.129/9)= .9589598905`	Displays the area of the curve to the right of the chi-square value. Proceed by pressing **[ENTER]**.
6.	`Exit Program?`	Enter **Y**.

You fail to reject the hypothesis when $Q(\chi^2)$ is in the range (0.025, 0.975) as it is in this example.

F-Distribution—"FDIS"

The F-statistic is the ratio of two variances. You may use this ratio to compare the variances of two normal populations. It may also be used to make inferences about the affect of one random variable on another by studying the population variance done in the "Analysis of Variance" section of this manual.

By definition, if X and Y are independent random variables having a chi-square distribution with df_1 and df_2 degrees of freedom, respectively, the F-statistic is

$$F_{df_1,df_2} = \frac{X/df_1}{Y/df_2}$$

with df_1 degrees of freedom used in the numerator and df_2 degrees of freedom used in the denominator. The following formulas compute the two needed degrees of freedom.

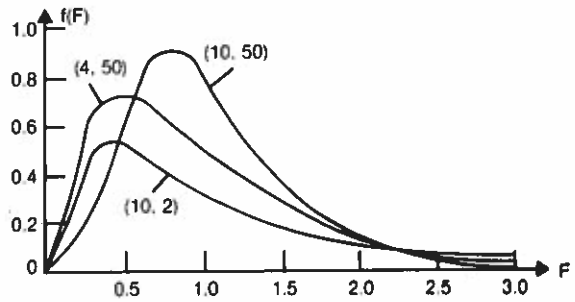
$$df_1 = K - 1$$
$$df_2 = N - K$$

where K is the number of samples and N is the total sample size. Like the t-Test curve, the shape of an F curve is determined by its number of degrees of freedom. The mean and variance are given by the following formulas.

$$\mu = df_2/(df_2 - 2)$$

$$\sigma^2 = \frac{2df_2^2\,(df_1 + df_2 - 2)}{df_1\,(df_2 - 2)(df_2 - 4)}$$

A random variable with an F distribution has no mean for $df_2 \leq 2$ and no variance when $df_2 \leq 4$. The following graph shows several F-distribution curves. Each curve is labeled with its degrees of freedom in the form of (df_1, df_2).



Method Used—F-Distribution Program

The tail area of the appropriate curve is calculated from one of the three formulas that follows. If df_1 is even, the formula is

$$Q(F:df_1, df_2) = x^{\frac{df_2}{2}}\left[1 + \frac{df_2}{2}(1-x) + \frac{df_2(df_2+2)}{2\times 4}(1-x)^2 + \ldots + \frac{df_2(df_2+2)\ldots(df_2+df_1-4)}{2\times 4\ldots(df_2-2)}(1-x)^{\frac{df_1-2}{2}}\right]$$

where

$X = df_2/(df_2 + df_1F)$

If df_2 is even, then

$$Q(F:df_1, df_2) = 1-(1-x)^{\frac{df_1}{2}}\left[1 + \frac{df_1}{2}x + \frac{df_1(df_1+2)}{2\times 4}x^2 + \ldots + \frac{df_1(df_1+2)\ldots(df_2+df_1-4)}{2\times 4\ldots(df_2-2)}x^{\frac{df_2-2}{2}}\right]$$

where x is defined above. The above formulas cover all cases except when both df_1 and df_2 are odd. This case is calculated by the following formula.

$$Q(F:df_1, df_2) = 1 - A(t:df_2) + \beta(df_1, df_2)$$

where

$$A(t:df_2) = \begin{cases} \frac{2}{\pi}\left(\theta + \sin\theta\left(\cos\theta + \frac{2}{3}\cos^3\theta + \ldots + \frac{2\times 4\ldots(df_2-3)}{3\times 5\ldots(df_2-2)}\cos^{df_2-2}\theta\right)\right) & \text{for } df_2 > 1 \\ \frac{2\theta}{\pi} \text{ for } df_2 = 1 \end{cases}$$

and

$$\beta(df_1, df_2) = \begin{cases} \frac{2}{\pi}\frac{\left(\frac{df_2-1}{2}\right)!}{\left(\frac{df_2-2}{2}\right)!}\sin\theta\cos^{1/2}\theta\left(1 + \frac{df_2+1}{3}\sin^2\theta + \ldots + \frac{(df_2+1)(df_2+3)\ldots(df_1+df_2-4)\sin^{df_1-3}\theta}{3\times 5\ldots(df_1-2)}\right) & \text{for } df_1 > 1 \\ 0 \text{ for } df_1 = 1 \end{cases}$$

and

$$\theta = \arctan\sqrt{\frac{df_1}{df_2}F}$$

Reference: *Handbook of Mathematical Functions*, Abramowitz and Stegun, National Bureau of Standards, 1964, p. 946.

User Instructions—F-Distribution

Select the F-Distribution program by entering **RUN "FDIS"**.

Step	Display	Procedure/Comment	Goto
1.	F-DISTRIBUTION	Program name.	2
2.	Use Printer?	a. Accept printer usage by entering **Y**.	3
		b. Reject printer usage by entering **N**.	4
3.	Enter Filename:	Enter output device name.	4
4.	Enter df1:	Enter the degrees of freedom (numerator).	5
5.	Enter df2:	Enter the degrees of freedom (denominator).	6
6.	Enter F:	Enter the F statistic.	7
7.	Q(###)=	Displays the area of the curve to the right of the F-statistic. Proceed by pressing **[ENTER]**.	8
8.	Exit Program?	a. Exit program by entering **Y**.	STOP
		b. Continue program by entering **N**.	4

Example 1

Suppose that you evaluated some data, using a one-way analysis of variance test, and found an F statistic of $F_{2,24} = 2.63$. Using an open-tailed test, determine whether you should reject your null hypothesis when testing at the 90% confidence level.

Select the F-Distribution program by entering **RUN "FDIS"**.

Step	Display	Procedure/Comment
1.	F-DISTRIBUTION	Program name.
2.	Use Printer?	Enter **N**.
3.	Enter df1: 0	Enter **2**.
4.	Enter df2: 0	Enter **24**.
5.	Enter F: 0	Enter **2.63**.
6.	Q(2/24)= .0927348639	Displays the area of the curve to the right of the F-statistic. Proceed by pressing **[ENTER]**.
7.	Exit Program?	Enter **Y**.

To test at the 90% confidence level, H_0 should be rejected if $P(F) > 0.90$. This is the same as rejecting H_0 when $Q(F) \leq 0.10$. Since $Q(2.63) < 0.10$ for 2 and 24 degrees of freedom, reject H_0.

Example 2

Find Q(F) for F 7,20 = 2.5. Then find P(F) for F 20,7 = 1/2.5 = 0.4. What do you notice about these results?

Select the F-Distribution program by entering **RUN "FDIS"**.

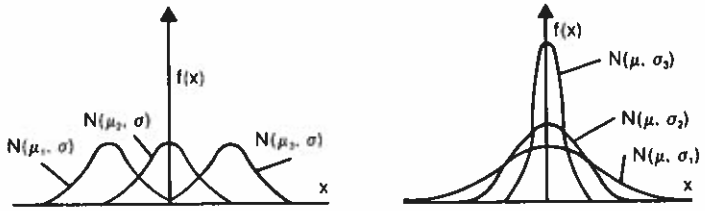
Step	Display	Procedure/Comment
1.	F-DISTRIBUTION	Program name.
2.	Use Printer?	Enter **N**.
3.	Enter df1: 0	Enter **7**.
4.	Enter df2: 0	Enter **20**.
5.	Enter F: 0	Enter **2.5**.
6.	Q(7/20)= .0510167666	Displays the area of the curve to the right of the F-statistic. Proceed by pressing **[ENTER]**.
7.	Exit Program?	Enter **N**.
8.	Enter df1: 7	Enter **20**.
9.	Enter df2: 20	Enter **7**.
10.	Enter F: 0	Enter **.4**.
11.	Q(20/7)= .9489832334	Displays the area of the curve to the right of the F-statistic. Proceed by pressing **[ENTER]**.
12.	Exit Program?	Enter **Y**.

This example illustrates what is known as the reciprocal property of the F-distribution, as illustrated by the following formula.

$Pr(F: df1,df2 \geq F) = Pr(F: df1,df2 \leq 1/F)$

Normal Distribution—"NORMAL"

The normal distribution is the most commonly used probability model in modern statistics. As a general rule, you may use this model whenever the distribution of the population you are sampling represents one of the curves illustrated below. However, for the best results, the parent population should have at least 100 elements, and the sample size should be greater than 30.



The diagram on the left illustrates three normal curves with the same standard deviation and different means. The second illustration shows three normal curves with the same mean and different standard deviations. The normal distribution actually has an infinite range. The density curve never reaches the x-axis even though it may appear to do so in the diagrams.

For the sake of convenience, a normal curve is said to be in standard form when it has a mean of zero and a standard deviation of one. Since any linear transformation of a normal variable yields a new normal variable, any normal variable x may be transformed to a measure called the standard deviate or z value. The z value is computed by using the following equation.

$$z = \frac{(x - \bar{x})}{\sigma}$$

Here $\bar{x}$ is the population mean, σ is its standard deviation, and z is the number of standard deviations that x is away from the mean.

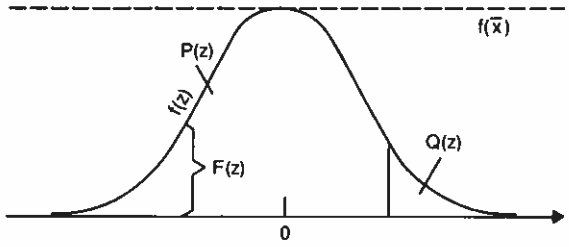
If you want to use the normal distribution model when your population is not normally distributed, you may use the central limit theorom. It will enable you to make inferences about the mean of your sample, x, if you are working with a large sample and know the mean and standard deviation of your parent population. Specifically, if n is your sample size,

$f(x) = z\ (\sqrt{n}\ (x - \bar{x})/\sigma)$.

The Normal Distribution program allows you to enter the standard deviate (z) and will compute either F(z), which is the height of the curve at that point, or Q(z), which is the area under the curve to the right of that point. You may also enter Q(z), and the program will compute the standard deviate. Note that the total area under curve f(z) equals 1; therefore, the area to the left of the standard deviate is computed by the formula

$P(z) = 1 - Q(z)$.

The following graph summarizes the concepts covered by the Normal Distribution program.



Analyzing With Small Samples

If you have a sample of five to ten elements or do not know the mean or standard deviation of your parent population, you should probably use the Student-t, the F, or the chi-square distribution. Statistics for small sample sizes often assume one of these distributions. Each of these distributions is related to the standard normal probability model. These relationships are based on the number of degrees of freedom of the sample distribution.

The number of degrees of freedom may be thought of as the number of free or functionally independent variables in a sample. For example, you cannot estimate the population variance with a one item sample. You need at least two variables to estimate the variance of a population. Even then, the variance can be based on only one item. Thus, one of the variables must be considered as functionally dependent on the other, and we say that the sample distribution has $2 - 1 = 1$ degree of freedom. In general, the number of degrees of freedom may be computed by the formula

$df = n - k$

where n is the number of items in the sample and k is the number of constraints present in the calculation of a given parameter.

The number of degrees of freedom affects the shape of a distribution curve, and the calculations for the degrees of freedom vary.

Method Used—Normal Distribution Program

$$F(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$$

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty e^{-\frac{t^2}{2}} dt$$

$$= F(z)\left(b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 + \epsilon(z)\right)$$

where

$\epsilon(z) < 7.5 \times 10^{-8}$

$t = \frac{1}{1 + pz}$

$p = .2316419$ $\quad b_3 = 1.781477937$

$b_1 = .319381530$ $\quad b_4 = -1.821255978$

$b_2 = -.356563782$ $\quad b_5 = 1.330274429$

To compute z, given $p = Q(z)$

$$z = t - \frac{c_0 + c_1 t + c_2 t^2}{1 + d_1 t + d_2 t^2 + d_3 t^3} + \epsilon(p)$$

where

$|\epsilon(p)| < 4.5 \times 10^{-4}$

$t = \sqrt{\ln(1/p^2)}$

$c_0 = 2.515517$ $\quad d_1 = 1.432788$

$c_1 = .802853$ $\quad d_2 = .189269$

$c_2 = .010328$ $\quad d_3 = .001308$

Reference: *Handbook of Mathematical Functions*, Abramowitz and Stegun. National Bureau of Standards, 1964, pp. 932-933.

User Instructions—Normal Distribution

Select the Normal Distribution program by entering **RUN "NORMAL"**.

Step	Display	Procedure/Comment	Goto
1.	NORMAL DISTRIBUTION	Program name.	2
2.	Use Printer?	a. Accept printer usage by entering **Y**.	3
		b. Reject printer usage by entering **N**.	4
3.	Enter Filename:	Enter output device name.	4
4.	0:Menu 1:Q(z) 2:F(Z) 3:z 4:Exit	Displays Menu.	5
5.	Enter Option:	Enter option number from Menu for unknown value to be computed.	
		0:Repeat Menu.	4
		1:Area of the normal curve to the right of the standard deviate.	6
		2:Height of the normal curve at the standard deviate.	8
		3:Standard deviate given area to the right of normal curve.	10
		4:Exit program.	STOP
6.	Enter z:	Enter the standard deviate.	7
7.	Q(###)=	Displays the area of the curve. Proceed by pressing **[ENTER]**.	5
8.	Enter z:	Enter standard deviate.	9
9.	F(###)=	Displays the height of the normal curve at z. Proceed by pressing **[ENTER]**.	5
10.	Enter Q(z):	Enter the area to the right of the normal curve.	11
11.	z=	Displays the standard deviate. Proceed by pressing **[ENTER]**.	5

Example

If the life of a flashlight battery is normally distributed with mean $\bar{x} = 120$ hours and a standard deviation $\sigma = 10$ hours, find the probability that the battery will last for more than 100 hours.

Select the Normal Distribution program by entering **RUN "NORMAL"**.

Step	Display	Procedure/Comment
1.	NORMAL DISTRIBUTION	Program name.
2.	Use Printer?	Enter **N**.
3.	0:Menu 1:Q(z) 2:F(Z) 3:z 4:Exit	Displays Menu.
4.	Enter Option: 0	Enter **1**.
5.	Enter z: 0	Enter **(100 – 120)/10**.
6.	Q(-2)= .977249938	Displays the area of the curve. Proceed by pressing **[ENTER]**.
7.	Enter Option: 1	Enter **4** to exit program.

In this example, x is first converted to a standard deviate using the expression

$$z = (x - \bar{x})/\sigma$$

Once this conversion is made, the probability of the battery lasting more than 100 hours is found as

$$Pr(Z > z) = Q(z).$$

Poisson Distribution—"POISS"

The Poisson distribution is useful when the probability of a specific event occurring is very small. The inspection and quality control of manufactured goods often yield data which exhibit a Poisson distribution since the number of defective articles in a large lot of goods should be small. The Poisson distribution is also used to approximate the binomial distribution when the number of trials, n, is large and the probability of occurrence, p, is small. The Poisson Distribution program can be used to calculate the probability of exactly k successes, f(k); the probability of k or fewer successes, P(k); and the probability of more than k successes, Q(k).

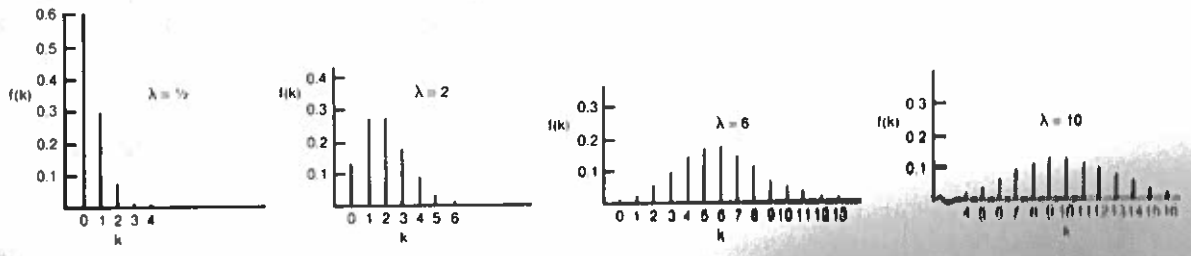
A term which must be defined for use in this program is the m parameter. The m parameter is the number of successes (k) which should occur in that sample given the sample size. The formula for the m parameter is given below.

m parameter = sample size × probability of success

or

$$m = \lambda \times t$$

where λ (lambda) represents the mean rate at which successes occur during the process, and t represents the period of time. Using the second formula, the m parameter represents the mean number of successes (k) which occur during time period t. Several Poisson distributions are illustrated below with various values for λ. The probability of exactly k successes is labeled as f(k).



Method Used—Poisson Distribution Program

$$f(k) = \frac{m^k e^{-m}}{k!}$$

where,

$m = np$

k = the number of successes

Reference: *Statistical Distributions*, Hastings and Peacock. Wiley and Sons, 1975, pp. 108-112.

User Instructions—Poisson Distribution

Select the Poisson Distribution program by entering **RUN "POISS"**

Step	Display	Procedure/Comment	Goto
1.	POISSON DISTRIBUTION	Program name.	2
2.	Use Printer?	a. Accept printer usage by entering **Y**.	3
		b. Reject printer usage by entering **N**.	4
3.	Enter Filename:	Enter output device name.	4
4.	Enter m:	Enter the m parameter.	5
5.	Enter k:	Enter the number of successes.	6
6.	f(k)=	Displays the probability of exactly k successes. Proceed by pressing **[ENTER]**.	7
7.	P(k)=	Displays the probability of k or fewer successes. Proceed by pressing **[ENTER]**.	8
8.	Exit Program?	a. Exit program by entering **Y**.	STOP
		b. Continue program by entering **N**.	4

Example 1

The number of accidents that occur in a busy intersection during a week are observed. They follow a Poisson distribution with m = 2.5. What is the probability that there will be 4 or more accidents at this intersection during a given week?

Select the Poisson Distribution program by entering **RUN "POISS"**.

Step	Display	Procedure/Comment
1.	POISSON DISTRIBUTION	Program name.
2.	Use Printer?	Enter **N**.
3.	Enter m: 0	Enter **2.5**.
4.	Enter k: 0	Enter **4**.
5.	f(4)= .1336018858	Displays the probability of exactly k successes. Proceed by pressing **[ENTER]**.
6.	P(4)= .8911780189	Displays the probability of k or fewer successes. Proceed by pressing **[ENTER]**.
7.	Exit Program?	Enter **Y**.

Q(k) is calculated by using the fact that Q(k) = 1 – P(k). This calculation is performed in BASIC by entering **PRINT 1 – .8911780189** resulting in .1088219811 and results in the probability of more than four accidents per week being 10.9%.

Example 2

Sheets of plastic are inspected and it is observed that the probability of spotting x defects in 10 minutes may be calculated by the formula for the Poisson distribution with m = 1.2. What is the probability of not finding any defects during 10 minutes of inspection?

Select the Poisson Distribution program by entering **RUN "POISS"**.

Step	Display	Procedure/Comment
1.	POISSON DISTRIBUTION	Program name.
2.	Use Printer?	Enter **N**.
3.	Enter m: 0	Enter **1.2**.
4.	Enter k: 0	Accept current value by pressing **[ENTER]**.
5.	f(0)= .3011942119	Displays the probability of exactly k successes. Proceed by pressing **[ENTER]**.
6.	P(0)= .3011942119	Displays the probability of k or fewer successes. Proceed by pressing **[ENTER]**.
7.	Exit Program?	Enter **Y**.

When considering the probability of 0 successes (k = 0), f(k) and P(k) are equal as seen above. During a 10 minute inspection, there is a 30% chance of not finding any defects.

Example 3

Use the Poisson distribution as an approximation of the binomial distribution to find the probability of obtaining 4 successes in 100 independent trials when p = 0.04.

Select the Poisson Distribution program by entering **RUN "POISS"**.

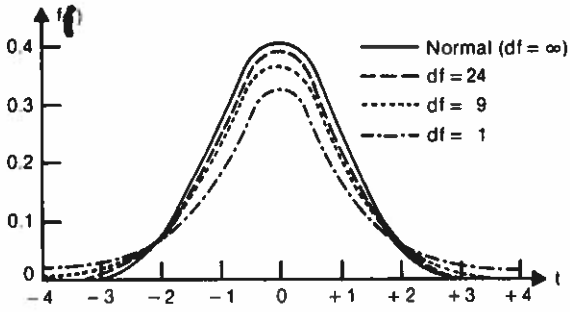
Step	Display	Procedure/Comment
1.	POISSON DISTRIBUTION	Program name.
2.	Use Printer?	Enter **N**.
3.	Enter m: 0	Enter **100*.04**.
4.	Enter k: 0	Enter **4**.
5.	f(4)= .1953668148	Displays the probability of exactly k successes. Proceed by pressing **[ENTER]**.
6.	P(4)= .6288369352	Displays the probability of k or fewer successes. Proceed by pressing **[ENTER]**.
7.	Exit Program?	Enter **Y**.

When n is large, the Poisson distribution can be used to approximate the binomial distribution. The estimated probability is 0.195. This approximation shows close agreement with the binomial distribution, where f(k) = 0.199.

Student's t-Distribution—"STUDENT"

When it can be assumed that the parent population is approximately normally distributed, t-curves are often used as probability models for small samples. The t-curve is most commonly used when the sample size is less than 30.

A t-curve with df > 1 degrees of freedom is similar to a standard normal curve because its mean is always zero. However, its standard deviation is given as $\sigma_n = df(df-2)$.* Notice that σ converges to one for large values of df. Therefore, if the degrees of freedom exceeds 30, the t-curve becomes the standard normal curve as illustrated below. When using this program, the methods of computing degrees of freedom vary so you must know the degrees of freedom for your sample.



* A random variable with t-distribution has no mean for df = 1 and no standard deviation for df ≤ 2.

Method Used—Student's t-Distribution Program

Because the Student's t-Distribution has a relationship to the F-distribution, the value P(t) is calculated from the following formula.

$$Pr((T:df) \le x) = \frac{1}{2}\left(1 + Pr((F:1,df) = x^2)\right)$$

Using this result, you may calculate the following probabilities manually

$Q(t) = 1 - P(t) = Pr(T \le t)$

$A(t) = 2P(t) - 1 = Pr(T \le |t|)$

where A(t) is the acceptance region when using a two-tailed test.

Reference: *Statistical Distributions*, Hastings & Peacock, Wiley, 1975, p. 120

User Instructions—Student's t-Distribution

Select the Student's t-distribution program by entering **RUN "STUDENT"**.

Step	Display	Procedure/Comment	Goto
1.	STUDENT'S t-DISTRIBUTION	Program name.	2
2.	Use Printer?	a. Accept printer usage by entering **Y**.	3
		b. Reject printer usage by entering **N**.	4
3.	Enter Filename:	Enter output device name.	4
4.	Enter df:	Enter the degrees of freedom.	5
5.	Enter t:	Enter the t-statistic.	6
6.	P(###)=	Displays the area of the curve to the left of the t-statistic. Proceed by pressing **[ENTER]**.	7
7.	Exit Program?	a. Exit program by entering **Y**.	STOP
		b. Continue program by entering **N**.	4

Example 1

An experimental engine has been previously tested for gasoline consumption. A short time later, it is decided to test the engine again to see if it is still operating at the same average gasoline consumption rate. Eight tests are run, and a t-statistic of 2.30 is obtained by comparing the new average consumption with the old average consumption. At a 95% confidence level, test the null hypothesis that the engine is operating at the same consumption rate.

Select the Student's t-distribution program by entering **RUN "STUDENT"**.

Step	Display	Procedure/Comment
1.	STUDENT'S t-DISTRIBUTION	Program name.
2.	Use Printer?	Enter **N**.
3.	Enter df: 0	Enter **7**.
4.	Enter t: 0	Enter **2.3**.
5.	P(2.3)= .9725044524	Displays the area of the curve to the left of the t-statistic. Proceed by pressing **[ENTER]**.
6.	Exit Program?	Enter **Y**.

You use a two-tailed test for this example since the new consumption rate may be significantly greater or significantly less than the old consumption rate. As a result, you fail to reject the null hypothesis if $0.025 \leq P(t) \leq 0.975$. Since $P(t) < .975$, you fail to reject your null hypothesis.

Example 2

Find the area under the t-curve with 30 degrees of freedom between $t = 0$ and $t = 1.2$. Compare this area with the corresponding area for the standard normal curve.

Select the Student's t-distribution program by entering **RUN "STUDENT"**.

Step	Display	Procedure/Comment
1.	STUDENT'S t-DISTRIBUTION	Program name.
2.	Use Printer?	Enter **N**.
3.	Enter df: 0	Enter **30**.
4.	Enter t: 0	Enter **1.2**.
5.	P(1.2)= .8802348246	Displays the area of the curve to the left of the t-statistic. Proceed by pressing **[ENTER]**.
6.	Exit Program?	Enter **N**.
7.	Enter df: 30.	Accept current value by pressing **[ENTER]**.
8.	Enter t: 1.2	Enter **0**.
9.	P(0)= .5	Displays the area of the curve to the left of the t-statistic. Proceed by pressing **[ENTER]**.
10.	Exit Program?	Enter **Y**.

This area is found by performing the following calculations.

$P(t \leq 1.2) - P(t \leq 0) = P(1.2) - P(0) = .8802348246 - .5 = .3802348264$

Executing the same calculations with the Normal Distribution program (using $P(z) = 1 - Q(z)$) results in Area = .3849302679. Note that the sizes of these areas are approximately the same, demonstrating the fact that the t-curve becomes the standard normal curve for $df \geq 30$.

Appendix—Subprograms

Many of the routines used in the programs of this library are developed as independent subprograms. These subprograms may be accessed for use in developing your own BASIC programs by calling the subprogram with its assigned arguments. Refer to the *Compact Computer User's Guide* for details on the restrictions and requirements for use of subprograms.

The special format used for presentation of subprograms in this appendix provides all of the information necessary for you to select and run the subprograms which you need in your own BASIC program. There are three possible sections within the discussion of each subprogram: Description, Format, and Example. To insure proper execution of the subprograms, these sections should be read thoroughly before running the subprograms.

Description	briefly describes the routine which the subprogram performs.
Format	illustrates the format necessary for calling the subprogram and identifies the requirements, restrictions, and purpose of each element in the argument list.
Example	provides the procedure and comments for actual execution of the subprogram.

BI

Description

Finds the probabilities associated with n trials of an experiment.

Format

CALL BI(N,P,K,F,PR)

N	Pass in the number of trials to be attempted.
P	Pass in the probability of successes.
K	Pass in the number of successes desired.
F	Pass in a dummy variable which returns the probability of exactly k successes.
PR	Pass in a dummy variable which returns the probability of k or fewer successes.

Example

```
N = 50
P = .5
K = 28
CALL BI(N,P,K,F,PR)
```

F is returned as .0788256707 and PR is returned as .8388818398.

CHI

Description

Finds the area to the right of the Chi-square.

Format

CALL CHI(DEGREES,CHISQR,OUTPUTS)

DEGREES	Pass in the degrees of freedom.
CHISQR	Pass in the chi-square statistic.
OUTPUT	Pass in a dummy variable which returns the area to the right of the chi-square statistic.

Example

```
DEGREES = 19
CHISQR = 24
CALL CHI(DEGREES,CHISQR,OUTPUTS)
```

OUTPUT is returned as .1961523586.

F

Description

Finds the height of the normal curve at the standard deviate.

Format

CALL F(Z,F)

Z	Pass in the value for the standard deviate.
F	Pass in a dummy variable which returns the height of the normal curve at the standard deviate.

Example

```
Z = .12
CALL F(Z,F)
```

F is returned as .3960802118

IR

Description

Input a number.

Format

CALL IR(TABPOS,PROMPT$,VARIABLE,DEVICE)

TABPOS	Position to tab to when printing the input value. Only used with a printer.
PROMPT$	Pass in a prompt describing the variable being asked for.
VARIABLE	Returns the value entered.
DEVICE	Pass in the output device number.

Example

CALL IR(15,"Standard Dev.",SD,PN)

1. The subprogram asks for input with the prompt

   ```
   Enter Standard Dev.: X
   ```

   where X is the current value of SD.

2. After input, the value of PN is checked. If it is 0, control returns to the calling program. If PN>0, the prompt, along with the entered number, are printed on device #PN.

   ```
   Standard Dev.= X
   ```

IZ

Description

Integer check greater than or equal to zero.

Format

CALL IZ(VARIABLE,FLAG)

VARIABLE Pass in number to be validated.

FLAG Returns a flag indicating if VARIABLE has failed a check by setting FLAG = −1 and passed by setting FLAG = 0. Prints an error message if FLAG = −1.

Example

CALL IZ(2.5,Flag)

Prints `VALUE MUST BE AN INTEGER.` Sets Flag = −1.

MN

Description

Finds the means and moments for paired or unpaired data in the form of one dimensional arrays.

Format

CALL MN(X( ),F( ),K,N,A,G,H,M1,M2,M3,F1,F2,TOTAL)

X( ) Pass in the values in the one dimensional array.

F( ) Pass in the values for the frequencies of the X values. If the data is ungrouped, pass F(0) = 0. If the data is grouped, pass F(0) = −2.

K Pass in the flag variable for multiple input. For single sets of data, pass K = 0. For multiple input, pass K = 0 on the first input. For successive inputs, pass K = 1. To end multiple input, pass K = −1.

N Pass in the number of elements in the array.

A Pass in a dummy variable which returns the arithmetic mean.

G Pass in a dummy variable which returns the geometric mean.

H Pass in a dummy variable which returns the harmonic mean.

M1 Pass in a dummy variable which returns the first moment.

M2 Pass in a dummy variable which returns the second moment.

M3 Pass in a dummy variable which returns the third moment.

F1 Flag variable returns −1 if invalid numbers are used to calculate harmonic mean. Otherwise returns 0.

F2 Flag variable returns −1 if invalid numbers are used to calculate the geometric mean. Otherwise returns 0.

TOTAL Pass in a dummy variable which returns the total number of observations.

NORM

Description

Finds the area to the right of the standard deviate for the normal distribution.

Format

CALL NORM(Z,Q)

Z	Pass in the value for the standard deviate.
Q	Pass in a dummy variable which returns the area to the right of the standard deviate.

Example

```
Z = -2
CALL NORM(Z,Q)
```

Q is returned as .977249938.

POIS

Description

Finds the probabilities associated with the Poisson distribution.

Format

CALL POIS(M,K,F,P)

M	Pass in the m parameter.
K	Pass in the number of successes.
F	Pass in a dummy variable which returns the probabilities of exactly k successes.
P	Pass in a dummy variable which returns the probabilities of k or fewer successes.

Example

```
M = 2.5
K = 4
CALL POIS(M,K,F,P)
```

F is returned as .1336018858 and P is returned as .8911780189.

SM

Description

Finds the sums and sums squared of rows for a two dimensional array.

Format

CALL SM(X(,),R,COL,SUM,SUMSQ)

X(,)	Pass in the values for a two dimensional array.
R	Pass in the value for the row to be summed.
COL	Pass in the value for the column to be summed.
SUM	Pass in a dummy variable which returns the sum of the elements in the given row.
SUMSQ	Pass in a dummy variable which returns the sum of the elements squared in the given row.

SORT

Description

Sorts, in ascending order, a one dimensional array using the Shell Sort method.

Format

CALL SORT(X( ),N)

X( )	Pass in the elements of a one dimensional array X( ).
N	Pass in the number of elements in the array.
X( )	Returns the sorted array in the one dimensional array X( )

Example

```
X(1) = 4
X(2) = 1
X(3) = 3
X(4) = 2
  N = 4
CALL SORT(X( ),N)
```

X( ) is returned as

X(1) = 1
X(2) = 2
X(3) = 3
X(4) = 4

UP

Description

Ask to use printer.

Format

CALL UP(NAME$,DEVICE)

NAME$	Pass in the name of program to be displayed for 3 seconds.
DEVICE	Returns the device number of the opened output device. A device number of 0 indicates output will appear on the display, while a device number of 1 indicates the requested output device was opened as device #1.

Example

```
CALL UP("NORMAL DISTRIBUTION",PN)
```

1. The name NORMAL DISTRIBUTION appears in the display for three seconds.
2. The subprogram then prompts

   Use Printer?

   An N response exits the subprogram with PN = 0.

   A Y response causes the subprogram to prompt

   Enter Filename:

   The entered device is opened under device number 1, setting PN = 1. The subprogram returns control to the calling program.

VAR

Description

Finds the area to the right of the f-statistic for the f-distribution.

Format

CALL VAR(DF1,DF2,F,Q)

DF1	Pass in the value for the degrees of freedom (numerator).
DF2	Pass in the value for the degrees of freedom (denominator).
F	Pass in the value for the F-statistic.
Q	Pass in a dummy variable which returns the area to the right of the F-statistic.

Example

```
DF1 = 2
DF2 = 24
  F = 2.63
CALL VAR(DF1,DF2,F,Q)
```

Q is returned as .0927348639.

WR

Description

Prints a space and then prints a message to the printer.

Format

CALL WR(MESSAGE$,PRINTER)

MESSAGE$	Pass in the message, in quotes, to be printed.
PRINTER	Pass in a number to the output device where the message is to be printed. If PRINTER = 0 then the message is not printed.

Example

```
CALL WR("MESSAGE",PRINTER)
```

The message MESSAGE is printed with a space preceding it.

YN

Description

Ask yes/no question.

Format

CALL YN(QUESTION$,ANSWER,DEVICE)

QUESTION$	Pass in the question to be asked with no question mark.
ANSWER	Returns ANSWER = 0 when N is chosen, or returns ANSWER = −1 when Y is chosen.
DEVICE	Pass in device number. If device number is zero, nothing is printed. If the device number is greater than zero, QUESTION$ is printed on the device.

Example

```
CALL YN("Edit",ANSWER,DEVICE)
```

The question Dbpo* is displayed.

A Y response sets ANSWER = −1 and prints the question, less the question mark, on device number DEVICE.

An N response sets ANSWER = 0. Control is then passed back to the calling program.

Z

Description

Finds the standard deviate of the normal curve given an area under the normal curve.

Format

CALL Z(Q,Z)

Q	Pass in the area under the normal curve.
Z	Pass in a dummy variable which returns the standard deviate.

Example

```
Q = .97725
CALL Z(Q,Z)
```

Z is returned as −2.000437772

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